Human Capital Risk, Public Consumption, and Optimal Taxation

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Abstract

This paper studies optimal tax policy when households invest in risky human capital and the government provides public services to households regardless of their economic status (”risk-free” public services). Specifically, we consider a tractable incomplete-market model with risk-less physical capital and risky human capital in which the government chooses a system of flat-rate taxes/subsidies as well as the level of spending on public services optimally. We use the tractability of the model to show theoretically that it is always socially optimal to subsidize investment in the risky asset, human capital. We also provide a quantitative analysis based on a version of the model calibrated to US data. Our main quantitative result is that implementing the optimal policy generates large growth and welfare gains if the labor-leisure choice is endogenous.

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1 Introduction

This paper is motivated by two empirical observations. First, there is strong evidence that human capital investment is risky, but complete insurance against this risk is lacking. More precisely, a significant fraction of labor income is the return to human capital investment, and a voluminous empirical literature has shown that individual households face large and highly persistent labor income shocks that have strong effects on individual consumption.[1] Second, in most developed countries, governments spend a significant amount of total output and a large fraction of this spending is used to provide public services to all households regardless of their economic status. In this paper, we show that both observations taken together imply that, in equilibrium, households always invest less in risky human capital than socially optimal. In other words, a subsidy to human capital investment financed through an incentive-neutral tax will improve social welfare. Moreover, we calibrate the model to the US data and show that the growth and welfare gains from implementing the optimal policy are large when the labor-leisure choice is endogenous.

There is a straightforward economic intuition for the sub-optimality of the equilibrium allocation without taxes and subsidies. In addition to the consumption-saving decision, households have to allocate their investment between a low-return, risk-free asset (physical capital in our model) and a high return, risky asset (human capital in our model), which in turn determines the mean and volatility of individual consumption growth. When making their portfolio decisions, individual households take aggregate variables, and in particular the level of output and public consumption services, as given. Thus, they do not take into account that more investment in the high-return asset, human capital, will increase aggregate output and, as shown in this paper, the optimal level of public consumption services. If private consumption and public consumption had the same degree of riskiness, then this would not pose a problem for the optimality of the market outcome. However, if public consumption is less risky than private consumption, then in equilibrium the private risk-return trade-off differs from the social risk-return trade-off, and

[1] For the estimation of income risk, see, for example, MaCurdy (1982), Carroll and Samwick (1997), Meghir and Pistaferri (2004), and Storesletten, Telmer, and Yaron (2004). For the consumption response, see, for example, Cochrane (1991), Flavin (1981), Townsend (1995), and Blundell, Pistaferri, and Preston (2008).

it becomes socially optimal to provide additional incentives for risk-taking.[2]

In this paper, we formalize the above intuition using a tractable endogenous growth model with incomplete markets. In our framework, households have the opportunity to invest in physical capital and human capital. While investment in physical capital is risk-free, human capital is subject to idiosyncratic depreciation shocks that are uninsurable and directly translate into permanent earning shocks. The government provides public services that are independent of idiosyncratic human capital shocks ("risk-free"), has access to a linear system of taxes and subsidies, and runs a balanced budget. For given government policy, our model is tractable in the sense that the equilibrium allocation can be characterized and computed without solving for the underlying wealth distribution. Using this tractability result, we characterize the optimal government policy and show theoretically the optimality of a human capital subsidy.

For the quantitative analysis, we calibrate the model to match a number of stylized facts for the US economy. We find that in the baseline model with fixed labor-leisure choice, the human-capital subsidy is substantial, but moving from the current US tax/subsidy system to the optimal system generates only small growth and welfare gains because of strong general equilibrium effects. More precisely, for given returns (partial equilibrium), an increase in human capital increases economic growth since human capital is the high-risk, high-return investment opportunity. In general equilibrium, this positive growth effect is dwarfed by a reduction in human capital returns since more human capital reduces the marginal product of labor, and our quantitative analysis reveals that the general – equilibrium effect is quite strong. However, once we allow for an endogenous labor-leisure choice, a corresponding increase in labor-time results in only small changes in the marginal product of labor despite a large increase in human capital, and the growth and welfare effects of moving to the optimal government policy therefore become large.

[2]From a different point of view, the presence of risk-free publicly provided consumption services works like an insurance system since the government applies a transfer scheme that shifts resources from risky private consumption to risk-free publicly provided consumption.

2 Related Literature

This paper is related to the extensive literature on optimal income taxation analyzing the so-called Ramsey problem (Judd (1985) and Chamley (1986)).[3] Most papers in this literature assume a representative household, but Aiyagari (1995), Davila, Hong, Ríos-Rull, and Krusell (2005), Conesa, Kitao, and Krüger (2009), and İmrohoroğlu (1998), have also considered the effect of uninsurable idiosyncratic risk and/or binding borrowing constraints. However, none of these papers allows for a risky investment opportunity. Recently, Panousi (2008) has analyzed optimal income taxation in a model with entrepreneurial risk, but she does not consider the endogenous choice of government spending. To the best of our knowledge, this is the first paper to study optimal taxation and government spending in an economy with idiosyncratic investment risk.[4]

There is also a theoretical literature that uses two-period models to study the welfare effects of income taxation when human capital investment is risky. In particular, Eaton and Rosen (1980) argue that linear labor income taxes may reduce human capital investment risk, so that it becomes optimal to tax labor income and simultaneously subsidize human capital investment to compensate for the des-incentive effect of the labor income tax. Clearly, in this paper we emphasize a very different economic mechanism.

3 The Model

This section develops the model that underlies the theoretical and quantitative analysis conducted in the subsequent sections. As in Krebs (2003), there is a competitive production sector using a production function that displays constant returns-to-scale with respect to the two input factors, physical capital and efficient labor. Households are ex-ante identical, infinitely-lived and have the opportunity to invest in physical and human capital. Investment in physical capital is risk-free, but investment in human capital is subject to idiosyncratic depreciation shocks. The government provides consumption services that enter directly the household's utility function,

[3]There is also an important strand of the literature analyzing optimal taxation in asymmetric-information economies (Mirrlees (1986) and Golosov, Kocherlakota and Tsyvinski (2003). For a recent review, see Kocherlakota (2005).

[4]Acemoglu and Zilibotti (1997) also consider a setting in which it is socially optimal to encourage risk taking, but their argument is very different from ours.

and levies linear taxes on capital, labor and consumption in order to satisfy a balanced budget constraint.

3.1 The Economy

Consider a discrete-time, infinite-horizon economy with one non-perishable good that can be either consumed or invested. Competition on the input factor markets and the neoclassical production technology allows to represent the production sector by an aggregate firm that takes factor prices as given. The aggregate firm uses physical capital K_t and efficiency units of labor L_tH_t to produce the *all-purpose* good. The production technology is given by $Y_t = F(K_t,\ L_tH_t)$, where L_t denotes hours worked, H_t human capital and thus, L_tH_t denotes efficiency units of hours worked. The rental rate of physical capital is r_{kt} and the rental rate of efficiency units of hours worked is r_{ht}. In each period, the firm hires capital and labor up to the point where current profits are maximized. Hence, the firm solves the following static maximization problem:

$$\max_{K_t,\ L_tH_t} \{F(K_t,\ L_tH_t) - r_{kt}\ K_t - r_{ht}\ L_t\ H_t\} \tag{1}$$

There are many ex-ante identical, infinitely-lived households with total mass of one. Households have identical preferences over private consumption plans $\{c_t\}_{t=0}^{\infty}$, private hours worked decisions $\{l_t\}_{t=0}^{\infty}$ and the sequence of publicly provided consumption services $\{G_t\}_{t=0}^{\infty}$. For convenience, let lower-case letters denote individual-specific variables and upper-case letters denote aggregate variables. The specification of the utility function closely follows Barro (1990) and Guo and Lansing (1999). The one period utility function is logarithmic, and with β denoting the time preference rate, expected lifetime utility is given by

$$U(\{c_t,\ l_t,\ G_t\}_{t=0}^{\infty}) = \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t (\log c_t + \nu_l\ \log(1-l_t) + \nu_g\ \log G_t)\right] \tag{2}$$

where ν_l and ν_g are utility parameters that measure how the household values labor and publicly provided consumption services.

Let k_t and h_t stand for the stock of physical and human capital owned by an individual household, and x_{kt} and x_{ht} denote the corresponding investment in physical and human capital. The fraction ϕ of human capital investment is bought with foregone earnings, whereas the fraction $(1-\phi)$ is directly bought by spending wealth. Physical capital investments are, as usual, completely bought with wealth. Capital and labor markets are perfectly competitive and the government taxes (or subsidizes) capital and labor income at the flat rate τ_{kt} and τ_{ht}. In addition, the government can tax consumption at rate τ_{ct} and for convenience, we define $\tau_t = (\tau_{kt},\ \tau_{ht},\ \tau_{ct})$. The sequential budget constraint reads

$$
\begin{aligned}
(1+\tau_{ct})\ c_t + x_{kt} + (1-\phi)\ x_{ht} &= (1-\tau_{kt})\ r_{kt}\ k_t + (1-\tau_{ht})(l_t\ r_{ht}\ h_t - \phi\ x_{ht}) \qquad (3)\\
k_{t+1} &= (1-\delta_k)\ k_t + x_{kt}, \quad k_t \geq 0\\
h_{t+1} &= (1-\delta_h+\eta_t)\ h_t + x_{ht}, \quad h_t \geq 0\\
&(k_0, h_0, \eta_0) \text{ given.}
\end{aligned}
$$

with δ_k and δ_h denoting the (average) depreciation rate of physical capital and human capital. The term η_t is a household-specific shock to human capital. We assume that these idiosyncratic shocks are identically and independently distributed across households and across time.[5] The random variable η_t represents uninsurable idiosyncratic labor income risk. A negative human capital shock, $\eta_t < 0$, can occur when a worker loses firm- or sector-specific human capital subsequent to job termination. In order to preserve the tractability of the model, the budget constraint rules out extended periods of unemployment because it assumes that wages are received in each period. Thus, the emphasis is on earnings uncertainty, not employment uncertainty. A decline in health provides a second example for a negative human capital shock. In this case, general and specific human capital might be lost. Internal promotions and upward movement in the labor market provide two examples of positive human capital shocks ($\eta_t > 0$). Clearly, human capital cannot become negative, restricting the domain of the shock distribution to $\eta_t \in (-(1-\delta_h), \infty)$.

Constraint (3) permits households to save ($x_{kt} > 0$) and dissave ($x_{kt} < 0$) at the going interest rate, but rules out the possibility of negative financial wealth ($k_t \geq 0$ and $h_t \geq 0$). Thus, one might conjecture that the equilibrium will change once households are allowed to accumulate debt. However, this is not the case for the model analyzed here, because income shocks are

[5] The budget constraint (3) makes two implicit assumptions about the accumulation of human capital. First, it lumps together general human capital (education and health) and specific human capital (on-the-job training). Second, (3) does not impose a non-negativity constraint on human capital investment ($x_{ht} \geq 0$).

permanent and not transitory and as shown in KUHN (2009), non-negativity constraints will never bind in such an environment. More precisely, the introduction of a risk-free bond does not change the equilibrium allocation as long as the bond interest rate, r_{bt}, is given by $r_{bt} = r_{kt} - \delta_k$.

For given initial state (k_0, h_0, η_0) and given fiscal policy, $\{\tau_t,\ G_t\}_{t=0}^{\infty}$, an individual household chooses a plan, $\{c_t,\ k_{t+1},\ h_{t+1},\ l_t\}_{t=0}^{\infty}$, that maximizes his expected lifetime utility (2) subject to the budget constraint (3). Clearly, in each period, the choice $(c_t,\ k_{t+1},\ h_{t+1},\ l_t)$ is a function of the history of idiosyncratic shocks, $\eta^t = (\eta_1,\ \dots,\ \eta_t)$.[6]

The budget constraint (3) can be rewritten in a way that shows that the households' optimization problem is a standard portfolio choice problem. To see this, define total wealth of an individual household as $w_t \doteq k_t + h_t$ and the fraction of total wealth invested in physical capital and human capital as $\theta_t \doteq k_t/w_t$ and $(1-\theta_t) \doteq h_t/w_t$, respectively. Using this notation, the budget constraint simplifies to

$$w_{t+1} = \frac{(1+r_t)\ w_t - (1+\tau_{ct})\ c_t}{\theta_{t+1} + (1-\phi\ \tau_{ht})\ (1-\theta_{t+1})} \tag{4}$$

$$w_t \geq 0,\ \ 0 \leq \theta_t \leq 1$$

$$(k_0, h_0, \eta_0) \text{ given.}$$

with the total investment return defined as

$$r_t \doteq \theta_t\ [(1-\tau_{kt})\ r_{kt} + (1-\delta_k)] + (1-\theta_t)\ [(1-\tau_{ht})\ l_t\ r_{ht} + (1-\delta_h+\eta_t)\ (1-\phi\ \tau_{ht})] - 1 \tag{5}$$

Equation (5) defines the return to investment as a function $r_t = r(\theta_t,\ l_t,\ \eta_t;\ r_{kt},\ r_{ht},\ \tau_{kt},\ \tau_{ht})$. Clearly, maximizing (2) subject to (3) with respect to $\{c_t,\ k_{t+1},\ h_{t+1},\ l_t\}_{t=0}^{\infty}$ is equivalent to maximizing (2) subject to (4) with respect to $\{c_t,\ w_{t+1},\ l_t,\ \theta_{t+1}\}_{t=0}^{\infty}$.

Finally, we assume that the government runs a balanced budget in each period which rules

[6] Observe that the tax system τ_t may depend on t, but not on idiosyncratic shocks η_t. In this sense, the tax system does not provide insurance against idiosyncratic human capitals shocks.

out extended periods of government debt. Thus, the government budget constraint reads

$$\tau_{ht}\left(r_{ht}\,\mathbb{E}[l_t(1-\theta_t)w_t] - \phi\,\mathbb{E}[(1-\theta_{t+1})w_{t+1} - (1-\delta_h+\eta_t)(1-\theta_t)w_t]\right) + \\ \tau_{kt}\,r_{kt}\,\mathbb{E}[\theta_t w_t] + \tau_{ct}\,\mathbb{E}[c_t] = G_t \quad (6)$$

From now on, we restrict the government to provide publicly consumption services proportional to the size of the economy. In particular, $G_t = \mu_t\,\mathbb{E}[c_t]$.[7]

3.2 Equilibrium

A competitive equilibrium of our model economy is defined as follows:

Definition 1 (Competitive Equilibrium)**.**
For any given initial distribution $(w_0,\ \theta_0,\ l_0)$, a competitive equilibrium is

1. *a sequence of $\{K_t, L_tH_t\}_{t=0}^{\infty}$ that solves the firm's maximization problem* (1) *for given factor prices $\{r_{kt},\ r_{ht}\}_{t=0}^{\infty}$;*
2. *a sequence of $\{c_t,\ w_{t+1},\ l_t,\ \theta_{t+1}\}_{t=0}^{\infty}$ that solves the household's optimization problem* (2) *subject to* (4) *for a given sequence of factor prices $\{r_{kt},\ r_{ht}\}_{t=0}^{\infty}$, idiosyncratic shocks $\{\eta_t\}_{t=0}^{\infty}$ and fiscal policy $\{\tau_t,\ \mu_t\}_{t=0}^{\infty}$, for all households;*
3. *a sequence of factor prices $\{r_{kt},\ r_{ht}\}_{t=0}^{\infty}$ that is consistent with market clearing on the input factor markets, $K_t = \mathbb{E}[\theta_t w_t]$ and $L_tH_t = \mathbb{E}[l_t(1-\theta_t)w_t]$; and*
4. *a sequence of fiscal policy $\{\tau_t,\ \mu_t\}_{t=0}^{\infty}$ that satisfies the government's balanced budget constraint* (6) *for given factor prices $\{r_{kt},\ r_{ht}\}_{t=0}^{\infty}$ and household policy $\{c_t,\ w_{t+1},\ l_t,\ \theta_{t+1}\}_{t=0}^{\infty}$, for all households.*

Introduce the aggregate capital to labor ratio $\tilde{K}_t \doteq \frac{K_t}{L_tH_t}$ and the production function in intensive form $f(\tilde{K}_t) \doteq F(\tilde{K}_t, 1)$. Using this notation, the first-order conditions associated with

[7] Taking BEA-data from 1970 to 2009, the government-to-household consumption ratio decreased from 29 percent in 1970 to 21 percent in 2000 and started to rise again to approximately 24 percent. In the long-run a constant ratio seems to be a good approximation.

the firm's static profit maximization problem (1) are

$$r_{kt} = f'(\tilde{K}_t)$$
$$r_{ht} = f(\tilde{K}_t) - \tilde{K}_t\ f'(\tilde{K}_t)$$

Thus, $r_{kt} = r_k(\tilde{K}_t)$, $r_{ht} = r_h(\tilde{K}_t)$. Note, in equilibrium, any sequence of factor prices is completely determined by a corresponding sequence of capital-to-labor ratios $\{\tilde{K}_t\}_{t=0}^{\infty}$. For convenience, we write $r(\theta_t,\ l_t,\ \eta_t;\ r_k(\tilde{K}_t),\ r_h(\tilde{K}_t),\ \tau_{kt},\ \tau_{ht}) = r(\theta_t,\ l_t,\ \eta_t;\ \tilde{K}_t,\ \tau_{kt},\ \tau_{ht})$.

We now discuss the households' optimization problem. The first-order conditions with respect to w_{t+1}, θ_{t+1} and l_t read

$$\frac{\theta_{t+1} + (1-\phi\ \tau_{ht})\ (1-\theta_{t+1})}{(1+\tau_{ct})\ c_t} = \beta\ \mathbb{E}\left[\frac{1 + r(\theta_{t+1},\ l_{t+1},\ \eta_{t+1};\ \tilde{K}_{t+1},\ \tau_{k,t+1},\ \tau_{h,t+1})}{(1+\tau_{c,t+1})\ c_{t+1}}\right] \tag{7}$$

$$\frac{\nu_l}{1-l_t} = (1-\theta_t)\ \frac{(1-\tau_{ht})\ r_h(\tilde{K}_t)}{(1+\tau_{ct})}\ \frac{w_t}{c_t} \tag{8}$$

$$\frac{\phi\ \tau_{ht}}{(1+\tau_{ct})\ c_t} = \beta\ \mathbb{E}\left[\frac{\hat{r}_k(\tilde{K}_{t+1},\ \tau_{k,t+1}) - \hat{r}_h(l_{t+1},\ \eta_{t+1};\ \tilde{K}_{t+1},\ \tau_{h,t+1})}{(1+\tau_{c,t+1})\ c_{t+1}}\right] \tag{9}$$

where

$$\hat{r}_{kt}(\tilde{K}_t,\ \tau_{kt}) = (1-\tau_{kt})\ r_k(\tilde{K}_t) + (1-\delta_k)$$
$$\hat{r}_{ht}(l_t,\ \eta_t;\ \tilde{K}_t,\ \tau_{ht}) = (1-\tau_{ht})\ l_t\ r_h(\tilde{K}_t) + (1-\delta_h+\eta_t)\ (1-\phi\ \tau_{ht})$$

denote the current returns to physical and human capital net of depreciation, respectively. The consumption-saving Euler equation (7) requires that the utility cost of saving one more unit of the *all-purpose* good must be equal to the expected discounted utility gain of doing so. The intratemporal first-order-condition (8) equates the marginal utility gain from leisure against the marginal benefit of working. Finally, the intertemporal first-order-condition (9) states that in the optimum, households are indifferent between investing one more unit into physical capital, on the one hand, and one more unit into human capital, on the other. Because of the assumption that idiosyncratic shocks are independently distributed over time, it suffices to take the unconditional expectation with respect to η_{t+1}.

Any plan $\{c_t,\ w_{t+1},\ l_t,\ \theta_{t+1}\}_{t=0}^{\infty}$ that is a solution to (7), (8), (9) and (4) and satisfies a corresponding transversality condition is also a solution to the utility maximization problem. Direct calculation shows that the consumption and saving policies

$$c_t = \frac{1-\beta}{1+\tau_{ct}}\ (1 + r(\theta_t,\ l_t,\ \eta_t;\ \tilde{K}_t,\ \tau_{kt},\ \tau_{ht}))\ w_t \tag{10}$$

$$w_{t+1} = \frac{\beta}{(\theta_{t+1} + (1-\phi\ \tau_{ht})\ (1-\theta_{t+1}))}\ (1 + r(\theta_t,\ l_t,\ \eta_t;\ \tilde{K}_t,\ \tau_{kt},\ \tau_{ht}))\ w_t \tag{11}$$

satisfy the household's budget constraint (4) and solve the consumption-saving Euler equation. Plugging the consumption and saving policies in the first-order-conditions with respect to hours worked (8), and solving for l_t finally yields the policy function for hours worked

$$\begin{aligned} l_t = &\frac{1}{(1+(1-\beta)\ \nu_l)} - \\ &\nu_l\ (1-\beta)\ \frac{\theta_t\ ((1-\tau_{kt})\ r_k(\tilde{K}_t) + (1-\delta_k)) + (1-\theta_t)\ (1-\delta_h+\eta_t)\ (1-\phi\ \tau_{ht})}{(1-\theta_t)\ (1-\tau_{ht})\ r_h(\tilde{K}_t)\ (1+(1-\beta)\ \nu_l)} \end{aligned} \tag{12}$$

Note that (12) defines a function $l_t = l(\theta_t,\ \eta_t;\ \tilde{K}_t,\ \tau_{kt},\ \tau_{ht})$. In particular, by the linearity of l_t in the idiosyncratic shock η_t, we find $\mathbb{E}[l(\theta_t,\ \eta_t;\ \tilde{K}_t,\ \tau_{kt},\ \tau_{ht})] = l(\theta_t,\ \mathbb{E}[\eta_t];\ \tilde{K}_t,\ \tau_{kt},\ \tau_{ht})$. Using the policy functions (10), (11) and (12), the first-order-condition with respect to the portfolio share θ_{t+1} simplifies to

$$\begin{aligned} &\frac{\phi\ \tau_{ht}}{(\theta_{t+1} + (1-\phi\ \tau_{ht})\ (1-\theta_{t+1}))} = \\ &\qquad \mathbb{E}\left[\frac{\hat{r}_k(\tilde{K}_{t+1},\ \tau_{k,t+1}) - \hat{r}_h(\theta_{t+1},\ \eta_{t+1};\ \tilde{K}_{t+1},\ \tau_{k,t+1},\ \tau_{h,t+1})}{1 + r(\theta_{t+1},\ \eta_{t+1};\ \tilde{K}_{t+1},\ \tau_{k,t+1},\ \tau_{h,t+1})}\right] \end{aligned} \tag{13}$$

Since the idiosyncratic shock η_{t+1} integrates out, the portfolio choice θ_{t+1} only depends on aggregate variables. Therefore, each household chooses the same capital share in his portfolio. The previously characterized plan $\{c_t,\ w_{t+1},\ l_t,\ \theta_{t+1}\}_{t=0}^{\infty}$ solves the set of the households' first-order conditions and it is straightforward to show that it also satisfies the associated transversality condition and, thus, is a solution to the utility maximization of an individual household. We summarize this result in the following proposition

Proposition 1 (Solution to the Household's Optimization Problem)**.**

Given the initial distribution over $(w_0,\ \theta_0,\ l_0)$*, for any given sequence of tax rates* $\{\tau_{ct},\ \tau_{kt},\ \tau_{ht}\}_{t=0}^{\infty}$ *and for any given sequence of capital-to-labor ratios* $\{\tilde{K}_t\}_{t=0}^{\infty}$*, the solution to the household's optimization problem is characterized as follows:*

1. *the optimal consumption and saving policy are linear homogenous in current wealth and explicitly given by* (10) *and* (11)*;*

2. *the optimal labor-leisure choice is independent of the household's wealth but depends on the current portfolio and current realization of the idiosyncratic shock; the policy function is explicitly given by* (12)*; and*

3. *the optimal portfolio choice is independent of the household's wealth and realization of the idiosyncratic shock; thus, every household chooses the same portfolio and* θ_{t+1} *is implicitly given as the solution to* (13).

Having characterized the households' decision problem, we now discuss the market clearing condition. In equilibrium, the aggregate capital-to-labor ratio has to be consistent with the investment choices of the households. By definition, $k_t = \theta_t w_t$ and $h_t = (1-\theta_t)w_t$ and since every agent chooses the same θ_{t+1} for $t \geq 0$, market clearing is given by

$$\tilde{K}_t^* = \begin{cases} \dfrac{\mathbb{E}[\theta_t\ w_t]}{\mathbb{E}[l(\theta_t,\ \eta_t;\ \tilde{K}_t^*,\ \tau_{kt},\ \tau_{ht})\ (1-\theta_t)\ w_t]} & \text{for } t = 0 \\ \dfrac{\theta_t}{l(\theta_t,\ \mathbb{E}[\eta_t];\ \tilde{K}_t^*,\ \tau_{kt},\ \tau_{ht})\ (1-\theta_t)} & \text{for } t > 0 \end{cases} \tag{14}$$

Since we allow for arbitrary initial distributions of wealth, portfolios and idiosyncratic shocks, we cannot guarantee mutual independence of $w_0,\ \theta_0$ and η_0. However, for $t > 0$, we know that θ_t and l_t are independent of wealth, and, moreover, every household chooses the same capital share in his portfolio. This allows us to simplify the market clearing condition for $t > 0$ further, as we already did in (14). The equilibrium capital-to-labor ratio $\tilde{K}_t^*$ is a fixed point to (14) for given portfolio choices and tax rates. The equilibrium path of the capital-to-labor ratio, $\{\tilde{K}_t^*\}_{t=0}^{\infty}$, is completely determined by the initial distribution over $(w_0,\ \theta_0,\ \eta_0)$, the complete sequence of capital and labor income taxes $\{\tau_{kt},\ \tau_{ht}\}_{t=0}^{\infty}$ and by the corresponding sequence of capital shares $\{\theta_{t+1}\}_{t=0}^{\infty}$ that solve the respective first-order condition of the household

$$\frac{\phi\ \tau_{ht}}{(\theta_{t+1} + (1-\phi\ \tau_{ht})\ (1-\theta_{t+1}))} = \mathbb{E}\left[\frac{\hat{r}_k(\tilde{K}^*_{t+1},\ \tau_{k,t+1}) - \hat{r}_h(\theta_{t+1},\ \eta_{t+1};\ \tilde{K}^*_{t+1},\ \tau_{k,t+1},\ \tau_{h,t+1})}{1 + r(\theta_{t+1},\ \eta_{t+1};\ \tilde{K}^*_{t+1},\ \tau_{k,t+1},\ \tau_{h,t+1})}\right]$$

More compactly, we write

$$IC(\theta_{t+1},\ \tilde{K}^*_{t+1},\ \tau_{ht},\ \tau_{k,t+1},\ \tau_{h,t+1}) = 0 \tag{15}$$

Note that this condition already uses the optimal policy functions for consumption, saving and leisure and thus satisfies the respective first-order condition and the household's budget constraint, automatically.

Using the households' plans as characterized in proposition 1 as well as the capital-to-labor ratio that satisfies market clearing, $\tilde{K}^*_t$, we find that the government budget constraint is independent of the current size of the economy. Again, using a more compact formulation, the government budget constraint reads

$$GC(\theta_t,\ \theta_{t+1},\ \tilde{K}^*_t,\ \tau_t,\ \mu_t) = 0 \tag{16}$$

For any initial distribution over $(w_0,\ \theta_0,\ \eta_0)$, and any given sequence $\{\theta_{t+1},\ \tau_{kt},\ \tau_{ht}\}_{t=0}^{\infty}$, the government can freely choose a sequence of consumption taxes and publicly provided consumption services $\{\tau_{ct},\ \mu_t\}_{t=0}^{\infty}$ that balances its budget out without distorting the equilibrium decisions of the firm and the households. Thus,

Proposition 2 (Competitive General Equilibrium)**.**
For any given initial distribution over $(w_0,\ \theta_0)$, the set of equilibria EQ is defined as

$$EQ = \Big\{ \{\theta_{t+1},\ \tau_t,\ \mu_t\}_{t=0}^{\infty} \mid \{\theta_{t+1},\ \tau_t,\ \mu_t\}_{t=0}^{\infty} \textit{ satisfies} \\ \{IC(\theta_{t+1},\ \tilde{K}^*_{t+1},\ \tau_{ht},\ \tau_{k,t+1},\ \tau_{h,t+1})\}_{t=0}^{\infty} \textit{ and } \{GC(\theta_t,\ \theta_{t+1},\ \tilde{K}^*_t,\ \tau_t,\ \mu_t)\}_{t=0}^{\infty} \Big\} \tag{17}$$

Proof. The proof is deferred to the appendix. □

In short, we can think of an equilibrium as a joint sequences $\{\theta_{t+1},\ \tau_t,\ \mu_t\}_{t=0}^{\infty}$ that satisfies the implementability constraint, (15), and the government's budget constraint, (16). Importantly, the equilibrium is independent of the actual wealth distribution.

4 Theoretical Results

For the derivation of the theoretical results, we use a simplified version of our model, which is specified in the following assumption:

Assumption 1.

1. *There is no disutility of work, $\nu_l = 0$, and households supply a fixed amount of hours worked that we conveniently normalize to unity, $l_t = 1,\ \forall t$.*

2. *Human capital is solely bought by spending the all-purpose good, $\phi = 0$.*

The second part of the assumption, $\phi = 0$, implies that the implementability and the government budget constraint simplify to

$$IC(\theta_{t+1},\ \tilde{K}^*_{t+1},\ \tau_{ht},\ \tau_{k,t+1},\ \tau_{h,t+1}) = IC(\theta_{t+1},\ \tilde{K}^*_{t+1},\ \tau_{k,t+1},\ \tau_{h,t+1})$$
$$GC(\theta_t,\ \theta_{t+1},\ \tilde{K}^*_t,\ \tau_t,\ \mu_t) = GC(\theta_t,\ \tilde{K}^*_t,\ \tau_t,\ \mu_t)$$

For any given sequence of capital and labor income tax rates, $\{\tau_{kt},\ \tau_{ht}\}_{t=0}^{\infty}$, proposition 1 specifies the equilibrium plan of consumption and wealth chosen by individual households. The particular representation of the equilibrium plan allows us to characterize the Pareto optimal equilibrium in a simple and transparent manner. Using the household's policy functions for a given fiscal policy $\{\tau_t,\ \mu_t\}_{t=0}^{\infty}$, expected lifetime utility from private consumption can be calculated as

$$\begin{aligned}\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t\ \log c_t\right] &= \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t\ \log\left(\frac{1-\beta}{1+\tau_{ct}}\ \beta^t\ \prod_{n=0}^{t}(1+r(\theta_n,\ \eta_n;\ \tilde{K}^*_n,\ \tau_{kn},\ \tau_{hn}))\ w_0\right)\right] \\ &= h(w_0) - \sum_{t=0}^{\infty} \beta^t\ \left(\log(1+\tau_{ct}) + \sum_{n=0}^{t} \mathbb{E}\left[\log(1+r(\theta_n,\ \eta_n;\ \tilde{K}^*_n,\ \tau_{kn},\ \tau_{hn}))\right]\right)\end{aligned}$$

where $h(w_0)$ is a function of the underlying model parameters and the initial wealth distribution. The welfare effect of any fiscal policy $\{\tau_t,\ \mu_t\}_{t=1}^{\infty}$ is independent of the initial wealth and asset distribution $(w_0,\ \theta_0)$. Similarly, the portfolio choices $\{\theta_{t+1}\}_{t=0}^{\infty}$ are independent of $(w_0,\ \theta_0)$, as well. Thus, any Pareto optimal equilibrium is preferred by all types of households, $(w_0,\ \theta_0)$, over all alternative equilibria, such that the set of Pareto optimal equilibria is independent of the initial distribution over household types. More precisely, any Pareto optimal equilibrium is the solution to the following constrained social planner problem[8]

$$\max_{\{\theta_{t+1},\ \tau_t,\ \mu_t\}_{t=0}^{\infty}} V\left(\{\theta_{t+1},\ \tau_t,\ \mu_t\}_{t=0}^{\infty}\right) \tag{18}$$

subject to

$$\{\theta_{t+1},\ \tau_t,\ \mu_t\}_{t=0}^{\infty} \in \Big\{ \{\theta_{t+1},\ \tau_t,\ \mu_t\}_{t=0}^{\infty} \mid \{\theta_{t+1},\ \tau_t,\ \mu_t\}_{t=0}^{\infty} \text{ satisfies}$$
$$\{IC(\theta_{t+1},\ \tilde{K}_{t+1}^{*},\ \tau_{k,t+1},\ \tau_{h,t+1})\}_{t=0}^{\infty} \text{ and } \{GC(\theta_t,\ \tilde{K}_t^{*},\ \tau_t,\ \mu_t)\}_{t=0}^{\infty} \Big\} \tag{19}$$

where the objective function in (18) is defined as[9]

$$\begin{aligned} V\left(\{\theta_{t+1},\ \tau_t,\ \mu_t\}_{t=0}^{\infty}\right) = (1+\nu_g) &\left(h(w_0) - \sum_{t=0}^{\infty} \beta^t \ \log(1+\tau_{ct}) \right) + \nu_g \sum_{t=0}^{\infty} \beta^t \ \log \mu_t \\ &+ \sum_{t=0}^{\infty} \beta^t \sum_{n=0}^{t} \mathbb{E}\left[\log(1 + r(\theta_n,\ \eta_n;\ \tilde{K}_n^{*},\ \tau_{kn},\ \tau_{hn})) \right] \\ &+ \nu_g \sum_{t=0}^{\infty} \beta^t \sum_{n=0}^{t} \log(1 + r(\theta_n,\ \mathbb{E}[\eta_n];\ \tilde{K}_n^{*},\ \tau_{kn},\ \tau_{hn})) \end{aligned}$$

The constrained social planner problem can be transformed into an unconstrained social planner problem as follows. Define

$$T_t = \theta_t \ \tau_{kt} \ r_{kt}(\tilde{K}_t^{*}) + (1-\theta_t) \ \tau_{ht} \ r_{ht}(\tilde{K}_t^{*})$$

which measures to what extend total investment is taxed ($T_t > 0$), respectively subsidized

[8]We use the terminology of constrained and unconstrained problems in the pure mathematical sense and do not associate them with the information structure in the economy, as it is often done in the literature.

[9]The objective function is calculated by plugging the households' policy functions for given fiscal policy successively in the life time utility as defined in (3).

($T_t < 0$). Using the new notation, the government budget constraint (6) can be written as

$$\tau_{ct} = \frac{\mu_t \ (1-\beta) \ (1 + r(\theta_t, \ \mathbb{E}[\eta_t]; \ \tilde{K}_t^*, \ 0, \ 0) - T_t) - T_t}{(1-\beta) \ (1 + r(\theta_t, \ \mathbb{E}[\eta_t]; \ \tilde{K}_t^*, \ 0, \ 0) - T_t) + T_t} \tag{20}$$

This defines a function $\tau_{ct} = \tau_c(\theta_t, \ \mu_t, \ T_t)$. For any choice $(\theta_t, \ \mu_t, \ T_t)$, the government budget constraint can be be satisfied by choosing τ_{ct} according to $\tau_c(\theta_t, \ \mu_t, \ T_t)$. Similarly, direct calculation shows that for any choice of $(\theta_t, \ \mu_t, \ T_t)$, the implementation constraint (15) will hold if capital and labor income taxes are

$$\begin{aligned} \tau_{h,t+1} &= -\left(\mathbb{E}\left[\frac{r_h(\tilde{K}_{t+1}^*)}{1 + r(\theta_{t+1}, \ \eta_{t+1}; \ \tilde{K}_{t+1}^*, \ 0, \ 0) - T_{t+1}}\right]\right)^{-1} \\ &\quad \times \ \mathbb{E}\left[\frac{\theta_{t+1} \ ((r_k(\tilde{K}_{t+1}^*) - \delta_k) - (r_h(\tilde{K}_{t+1}^*) - \delta_h + \eta_{t+1})) - T_{t+1}}{1 + r(\theta_{t+1}, \ \eta_{t+1}; \ \tilde{K}_{t+1}^*, \ 0, \ 0) - T_{t+1}}\right] &(21) \\ \tau_{k,t+1} &= \frac{T_{t+1} - (1 - \theta_{t+1}) \ r_k(\tilde{K}_{t+1}^*) \ \tau_{h,t+1}}{\theta_{t+1} \ r_k(\tilde{K}_{t+1}^*)} &(22) \end{aligned}$$

This defines functions $\tau_{ht} = \tau_h(\theta_t, \ \mu_t, \ T_t)$ and $\tau_{kt} = \tau_k(\theta_t, \ \mu_t, \ T_t)$. The constrained social planner problem (18) subject to (19) is equivalent to the unconstrained social planner problem

$$\max_{\{\theta_{t+1}, \ \mu_t, \ T_t\}_{t=0}^{\infty}} \tilde{V}\left(\{\theta_{t+1}, \ \tau_t, \ \mu_t\}_{t=0}^{\infty}\right) \tag{23}$$

where

$$\begin{aligned} \tilde{V}\left(\{\theta_{t+1}, \ \mu_t, \ T_t\}_{t=0}^{\infty}\right) &= (1+\nu_g) \ \left(h(w_0) - \sum_{t=0}^{\infty} \ \beta^t \ \log(1 + \tau_c(\theta_t, \ T_t, \ \mu_t))\right) + \nu_g \ \sum_{t=0}^{\infty} \ \beta^t \ \log \mu_t \\ &\quad + \sum_{t=0}^{\infty} \ \beta^t \ \sum_{n=0}^{t} \ \mathbb{E}\left[\log(1 + r(\theta_n, \ \eta_n; \ \tilde{K}_n^*, \ 0, \ 0) - T_n)\right] \\ &\quad + \nu_g \ \sum_{t=0}^{\infty} \ \beta^t \ \sum_{n=0}^{t} \ \log(1 + r(\theta_n, \ \mathbb{E}[\eta_n]; \ \tilde{K}_n^*, \ 0, \ 0) - T_n) \end{aligned}$$

The discussion above is summarized in the following proposition:

Proposition 3 (Equivalence of Constrained and Unconstrained Planner Problem)**.**
Any Pareto optimal equilibrium allocation can be found by solving either the constrained social planner problem, (18) *subject to* (19)*, or the unconstrained social planner problem,* (23)*.*

Straightforward but tedious calculations reveal that the objective function in (23) is strictly concave. Since there is a convex choice set, the social planner problem (23) has at most one solution. In the Appendix we show that the maximization problem has indeed a solution. Thus, there is a unique solution to the social planner problem (23), and therefore a unique Pareto optimal equilibrium. As proofed it the appendix, the solution to the social planner's problem has the following properties:

Proposition 4 (Optimal Taxes and Public Services)**.**
Let $\{\theta_t,\ \mu_t,\ T_t\}_{t=0}^{\infty}$*, respectively* $\{\theta_t,\ \mu_t,\ \tau_{ct},\ \tau_{kt},\ \tau_{ht}\}_{t=0}^{\infty}$*, be the solution to the social planner problem. If* $\sigma_\eta > 0$ *and* $\nu_g > 0$*, the solution to the social planner problem is characterized by:*

(i.) Optimality of a stationary fiscal policy

$$(\theta_t,\ \mu_t,\ T_t) = (\theta,\ \mu,\ T)$$

and in particular

$$(\theta_t,\ \mu_t,\ \tau_{ct},\ \tau_{kt},\ \tau_{ht}) = (\theta,\ \mu_g,\ \tau_c,\ \tau_k,\ \tau_h)$$

(ii.) Optimality level of government spending

$$\mu = \nu_g$$

(iii.) Optimality of subsidizing total investment

$$T < 0$$

(iv.) Optimality of subsidizing human capital

$$\tau_h < 0$$

5 Quantitative Analysis

5.1 Calibration

For our quantitative analysis, we use 3 different specifications of our model economy. We calibrate the models such that the stationary equilibrium is consistent with stylized annual facts of the US economy. In order to preserve the comparability of our results, we re-calibrate the model for each specification. The specifications are as follows: First, households do not value leisure, $\nu_l = 0$, and we normalize their labor supply to unity, $l_{it} = 1$. Furthermore, investment is solely bought by spending wealth, meaning $\phi = 0$. This is basically the model setup for which we derived the theoretical results in the previous section. In the second specification, households value leisure, but there is still no human capital investment through forgone earnings. Finally, third, we impose the restriction that a fraction $\phi = 0.25$ of human capital investment is bought through foregone earnings. This parameter value is consistent with the range of values found and applied by Trostel (1993).

Because the main contribution of our paper is to provide an argument that the optimal tax rates are distortionary to the portfolio decision of the households, we choose an economic environment without distortionary taxes as benchmark to which we calibrate the relevant model parameters. Thus, $\tau_k = \tau_h = 0$.

We now calibrate the preference parameters. Since we are not interested in the welfare effect of changes in the public good provision, we assume that the government already provides the optimal amount of public consumption services. The optimality condition with respect to public service provision is $\nu_g = \mu$.[10] Since $\mu = \frac{g_t}{\mathbb{E}[c_t]}$, we simply set ν_g to the average government consumption to private consumption ratio found in US time series. Thus, $\nu_g = 0.255$ and in order to satisfy the government budget constraint, we moreover get $\tau_c = 0.255$, as well.[11] As noted previously, the first model specification sets $\nu_l = 0$. For the second and third model specification, we choose ν_l such that the average labor supply in equilibrium amounts to one third of the households' time endowment. For both specifications, we find $\nu_l = 2.5568$. Finally, the time preference rate is set such that the equilibrium saving rate is 20 percent. For the first

[10] Actually, in section 3, we have shown that this condition holds for the first model specification. However, it is straightforward to prove that this condition also extends to the more elaborated specifications two and three.

[11] The government and private consumption time series includes years 1970 to 2009 and is taken from the BEA NIPA tables.

specification, we get $\beta = 0.9247$ while specification two and three require $\beta = 0.9250$.

The depreciation rates are set to $\delta_k = \delta_h = 0.06$. For physical capital, this value lies within the range suggested by the literature, e.g. Cooley and Prescott (1995). For human capital, Browning, Hansen and Heckman (1999) find annual depreciation rates between 0 and 4 percent. Accounting for the infinite horizon structure in our model, we have to add an additional depreciation of 2 percent to capture full depreciation of human capital after 50 years of work life. Thus, $\delta_h = 0.06$ is at the upper bound of reasonable values suggested in the literature. For the i.i.d. depreciation shock to human capital, we assume that $\eta \sim N(\mu,\ \sigma,\ \underline{a},\ \overline{a})$ with $\underline{a}$ and $\overline{a}$ denoting the lower and upper truncation point of the distribution. We set $\underline{a} = -0.75$ and $\overline{a} = 0.75$ which guarantees the minimum requirement on the domain of the η-distribution, $\eta > -(1 - \delta_h)$ and $\mathbb{E}[\eta] = 0$. Clearly, η is a permanent human capital shock which translates into a permanent labor income shock for the household. The evolution of logged labor income is governed by

$$\begin{aligned}\log h_{i,t+1} &= \log(\beta\ [1 + r(\theta,\ \eta_t;\ \tilde{K}^*,\ \tau_k,\ \tau_h)\ h_t]) \\ &\approx \log\beta + \log h_{it} + r(\theta,\ \eta_t;\ \tilde{K}^*,\ \tau_k,\ \tau_h) \\ &= \omega + \frac{(1-\theta)\ (1-\phi\ \tau_h)}{1+(1-\beta)\ \nu_l}\ \eta_t\end{aligned}$$

with some constant ω which contains all terms that do not include the idiosyncratic human capital depreciation shock. The logarithm of labor income follows approximately a random walk with drift. And the mean and the standard deviation of the permanent component of logged labor income are $\mu_{yh} = 0$ and $\sigma_{yh} = \frac{(1-\theta)\ (1-\phi\ \tau_h)}{1+(1-\beta)\ \nu_l}\ \sigma_\eta$. In the empirical literature, the random walk specification is often used to model the permanent component of labor income risk. For example, Carroll and Samwick (1997) find a standard deviation of 0.147 whereas Meghir and Pistaferri (2004) estimate a value of 0.182. Storesletten, Telmer and Yaron (2004) additionally condition labor income risk on the business cycle and find that labor income risk varies between 0.12 and 0.21. We calibrate σ_η such that in equilibrium, permanent labor income risk exhibits a standard deviation of 0.15. This yields $\sigma_\eta = 0.2532$ in the first specification and $\sigma_\eta = 0.3013$ in the second and third specification.

The aggregate production technology is Cobb-Douglas with intensive form representation

$f(\tilde{K}) = z\tilde{K}^{\alpha}$. We set $\alpha = 0.36$ to match the capital share of income according to the values suggested in the literature. The technology parameter z is chosen such that in equilibrium, aggregate consumption grows at 2 percent per annum. For the first specification, this yields $z = 0.3184$ and for the second and third specification, we get $z = 0.6386$. The calibration values are given in table 1, while the associated equilibrium allocations can be read off from table 2.

Table 1: Calibration

	DESCRIPTION	(1)	(2)	(3)	MATCHES
α	technology parameter	0.3600	0.3600	0.3600	capital share
δ_k, δ_h	depreciation rates	0.0600	0.0600	0.0600	
ϕ	forgone earnings	0	0	0.2500	
ν_g	utility parameter	0.2550	0.2550	0.2550	government expenditure to private consumption ratio
z	technology parameter	0.3192	0.6442	0.6442	consumption growth rate 2%
β	time preference rate	0.9230	0.9231	0.9231	saving rate 20%
σ	sd depreciation shock	0.2602	0.3269	0.3269	labor income risk 0.15
ν_l	utility parameter	0	2.4851	2.4851	labor supply $\mathbb{E}[l_i] = 1/3$

5.2 Results

The optimal tax policy and its welfare and growth implications are given in table 2. Welfare effects ΔW are computed according to Lucas (1987). In the first model specification, the optimal capital and labor income tax rates are 2.0 and -2.5 percent, respectively. Due to a very strong general equilibrium effect, the possibility to encourage more risk taking is limited. Specifically, reducing the labor income tax leads to a portfolio shift from physical to human capital. Consequently, the equilibrium capital-to-labor ratio decreases thereby pushing the equilibrium interest rate upwards and the equilibrium wage rate downwards and thus discourages human capital investment. Taken together, the reduction of the tax rate on labor income induces a decrease in the wage rate such that the effect of the tax policy on the net return to human capital is almost offset. Clearly, following this line of argument, there is only a small spread between the optimal tax rates leading to minor welfare gain of about 0.06 percent from implementing the optimal tax system. Of course, in a small open economy where prices are exogenously fixed by the world financial market, the general equilibrium effect would be absent leading to more substantial welfare effects. In this sense, our framework establishes a lower bound of the welfare effects.

Adding an endogenous labor-leisure choice helps to break the strong general equilibrium effect. Now, reducing the labor income tax rate encourages both, investment into human capital and labor supply. Clearly, investing into human capital becomes more profitable if the household simultaneously increases his labor supply. The opposite holds as well: raising labor supply makes investment into human capital more profitable. This reinforcing effect helps to overrule the general equilibrium effect more easily and for the calibrated model economy, the optimal labor income tax drops to -9.5 percent whereas the optimal capital income tax rises to 5.4 percent. The established spread between both tax rates of 14.9 percentage points leads to substantial welfare gains of 1.49 percent and the annual growth rate rises by substantial 0.67 percentage points.

Imposing that 25 percent of human capital investment are payed by foregone earnings leads to stronger reduction of the optimal labor income tax compared to the previous result. This result is due to the fact that the fraction of labor income that is invested through foregone earnings is exempted from the labor income tax. Thus, subsidizing human capital needs to take into account that a fraction of the subsidy is basically not payed out and therefore raises labor income by a lower amount as in specification two. Put differently, the government has to rise labor income subsidy beyond the previous result in order to encourage sufficient risk taking. The spread between capital and labor income tax increases to 17 percent, the welfare gain however is slightly lower as before (because the policy is not as effective as previously). The same holds for the growth effect.

Finally, we allow the government to encourage human capital investment more directly by subsidizing the investment into human capital. Clearly, the direct policy dominates the indirect policy. It even happens, that it is now optimal to tax labor income in order to subsidize investment into human capital. The net effect on human capital accumulation, however, remains positive, as can be seen from the drop of the equilibrium portfolio choice from 0.4124 to 0.3600.

Table 2: Results

	benchmark	(1)	(2)	(3a)	(3b)
OPTIMAL ALLOCATION					
θ	0.4127 (0.4124)	0.4024	0.3751	0.3781	0.3600
$E[l_i]$	1 (1/3)	1	0.3645	0.3657	0.3451
OPTIMAL POLICY					
τ_k	0	0.0198	0.0540	0.0510	0.0695
τ_h	0	-0.0249	-0.0950	-0.1188	0.1024
τ_c	0.2550	0.2768	0.3702	0.3869	0.2704
σ_h	0	0	0	0	0.2465
WELFARE/ GROWTH EFFECTS (IN PERCENT/ PERCENTAGE POINTS)					
$\triangle W$		0.06	1.49	1.39	1.91
$\triangle \gamma_c^*$		0.14	0.67	0.62	0.80

Thus, the government still wants to encourage more risk taking by the private households. The optimal capital income tax and the optimal labor income tax are 7.0 and 10.2 percent, respectively, and human capital investment is now subsidized by 24.7 percent. Of course, giving the government one more instrument, it cannot do worse. The welfare effect rises to substantial 1.9 percent while the annual growth rate increases by substantial 0.8 percentage points.

6 Conclusions

We showed that under the very plausible assumptions that first, there is a substantial amount of idiosyncratic risk and second, governments provide a significant amount of non-wasteful consumption services, there is too less investment into the risky asset in the competitive equilibrium. The social planner can thus implement a welfare improving tax policy that encourages more risk taking by the households. However, there are strong general equilibrium effects that may easily overrule the benefits of subsidizing the risky asset, leading to very small welfare and growth effects of the optimal policy. However, in our model with human capital risk, introducing an endogenous labor leisure choice substantially breaks the strong general equilibrium effect.

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Appendix

Proof of Proposition 4

For convenience, we rewrite

$$\begin{aligned}
r_t &= \theta_t\ [(1-\tau_{kt})\ r_{kt} - \delta_k] + (1-\theta_t)\ [(1-\tau_{ht})\ l_t\ r_{ht} - \delta_h + \eta_t] \\
&= \theta_t\ [r_{kt} - \delta_k] + (1-\theta_t)\ [l_t\ r_{ht} - \delta_h] - \theta_t\ \tau_{kt}\ r_{kt} - (1-\theta_t)\ \tau_{ht}\ r_{ht} + (1-\theta_t)\ \eta_t \\
&= \Big\{\theta_t\ [r_{kt} - \delta_k] + (1-\theta_t)\ [l_t\ r_{ht} - \delta_h + \eta_t]\Big\} - T_t + (1-\theta_t)\ \eta_t
\end{aligned}$$

and define the term in curly brackets as $\bar{r}(\theta_t, \cdot)$. Thus,

$$r_t = \bar{r}(\theta_t, \cdot) - T_t + (1-\theta_t)\ \eta_t$$

With this notation, we now proceed with the proof of the proposition.

Step 1: the social planner's first-order condition

The first order conditions of the social planner problem read

$$\frac{\partial \tilde{V}(\{\theta_t,\ \mu_t,\ T_t\})}{\partial \mu_t} = -\frac{1+\nu_g}{1+\mu_t} + \frac{\nu_g}{\mu_t} = 0 \tag{24}$$

$$\begin{aligned}
\frac{\partial \tilde{V}(\{\theta_t,\ \mu_t,\ T_t\})}{\partial T_t} &= \mathbb{E}\left[\frac{1}{1+\bar{r}(\theta_t,\ \cdot) - T_t + (1-\theta_t)\eta_t}\right] \\
&\quad - \frac{(1+\nu_g)\ (1-\beta)\ \beta}{(1-\beta)(1+\bar{r}(\theta_t,\ \cdot) - T_t) + T_t} - \frac{(1-\beta-\nu_g\beta)}{1+\bar{r}(\theta_t,\ \cdot) - T_t} = 0
\end{aligned} \tag{25}$$

$$\begin{aligned}
\frac{\partial \tilde{V}(\{\theta_t, \mu_t,\ T_t\})}{\partial \theta_{t+1}} &= \mathbb{E}\left[\frac{(r_k(\theta_{t+1}) - \delta_k) - (r_h(\theta_{t+1}) - \delta_h + \eta_{t+1})}{1+\bar{r}(\theta_{t+1},\ \cdot) - T_{t+1} + (1-\theta_{t+1})\eta_{t+1}}\right] \\
&\quad + \frac{(1+\nu_g)\ (1-\beta)\ (1-\beta)\ ((r_k(\theta_{t+1}) - \delta_k) - (r_h(\theta_{t+1}) - \delta_h))}{(1-\beta)(1+\bar{r}(\theta_{t+1},\ \cdot) - T_{t+1}) + T_{t+1}} \\
&\quad - \frac{(1-\beta-\nu_g\beta)(r_k(\theta_{t+1}) - \delta_k) - (r_h(\theta_{t+1}) - \delta_h)}{1+\bar{r}(\theta_{t+1},\ \cdot) - T_{t+1}} = 0
\end{aligned} \tag{26}$$

Step 2: proof of part (i.) and (ii.)

Updating equation (25) and combining it with equation (26) reveals, that (θ_{t+1}, T_{t+1}) only depends on period $t+1$ variables and thus, $(\theta_{t+1}, T_{t+1}) = (\theta, T),\ \forall t$. Moreover, condition (24) immediately yields $\mu_t = \nu_g,\ \forall t$. Taken together, this proofs part (i.) of the proposition, $(\theta_t,\ T_t) = (\theta,\ T)$ which implies, due to the tax functions (20) to (21), $(\theta_t,\ \mu_t,\ \tau_{ct},\ \tau_{kt},\ \tau_{ht}) = (\theta,\ \mu_g,\ \tau_c,\ \tau_k,\ \tau_h)$, and part (ii.) of the proposition, $\mu_t = \nu_g$.

Step 3: proof of part (iii.)

Updating (25) and combining it with (26) yields

$$\mathbb{E}\left[\frac{\eta}{1+\bar{r}(\theta,\ \cdot)-T+(1-\theta)\eta}\right] = (1+\nu_g)(1-\beta)\frac{(r_k(\theta)-\delta_k)-(r_h(\theta)-\delta_h)}{(1-\beta)(1+\bar{r}(\theta,\ \cdot)-T)+T} \tag{27}$$

where we dropped the time indices for convenience. It is easier to work with (27) instead of (26). For simplicity, define

$$\Omega_T(\theta,\ T) = \mathbb{E}\left[\frac{1}{1+r(\theta,\ \cdot)-T+(1-\theta)\eta}\right] - \frac{(1+\nu_g)\ (1-\beta)\ \beta}{(1-\beta)(1+\bar{r}(\theta,\ \cdot)-T)+T} - \frac{1-\beta-\nu_g\beta}{1+\bar{r}(\theta,\ \cdot)-T} \tag{28}$$

$$\Omega_\theta(\theta,\ T) = \mathbb{E}\left[\frac{\eta}{1+\bar{r}(\theta,\ \cdot)-T+(1-\theta)\eta}\right] - (1+\nu_g)(1-\beta)\frac{(r_k(\theta)-\delta_k)-(r_h(\theta)-\delta_h)}{(1-\beta)(1+\bar{r}(\theta,\ \cdot)-T)+T} \tag{29}$$

We now have to show that there exists a (θ^*, T^*) that solves $\Omega_T(\theta^*,\ T^*) = 0$ and $\Omega_\theta(\theta^*,\ T^*) = 0)$.

First, we show that for any $\theta \in [0,1]$, $\exists\ T \in [\underline{T}, 0]$ that solves equation (28). Fix $\theta = \bar{\theta}$ and $\underline{T} = -\frac{1-\beta}{\beta}(1+\bar{r}(\bar{\theta},\ \cdot))$. Since $\bar{r}(\bar{\theta},\ \cdot)$ is finite for any $\bar{\theta}$, the lower bound $\underline{T}$ is well defined. We now apply the intermediate value theorem. On the one side, taking the limit $\lim_{T\searrow\underline{T}} \Omega_T(\bar{\theta}, T)$, we find that the first and the third term are finite while the second term goes to infinity. Thus, $\lim_{T\searrow\underline{T}} \Omega_T(\bar{\theta}, T) = -\infty < 0$. on the other sides, we evaluate $\Omega_T(\bar{\theta}, T)$ at $t = 0$ which yields

$$\Omega_T(\bar{\theta}, 0) = \mathbb{E}\left[\frac{1}{1+\bar{r}(\bar{\theta})+(1-\theta)\eta}\right] - \frac{1}{1+\bar{r}(\theta,\ \cdot)}$$

Due to strict convexity of $\frac{1}{1+r(\bar{\theta})+(1-\theta)\eta}$ in η, Jensen's inequality leads to

$$\Omega_T(\bar{\theta}, 0) \geq \frac{1}{1+\bar{r}(\theta,\ \cdot)} - \frac{1}{1+\bar{r}(\theta,\ \cdot)} = 0$$

Thus, $\Omega_T(\bar{\theta}, 0) \geq 0$. Clearly, $\Omega_T(\bar{\theta}, T)$ is continuous on the domain of T such that applying the intermediate value theorem implies that for any $\theta \in [0, 1]$, there exists a $T \in [-\frac{1-\beta}{\beta}(1+\max \bar{r}(\theta,\ \cdot)), 0]$ that solves the first order condition with respect to T. Moreover, the map $T = T(\theta)$ is continuous.

Next, we show that for the continuous map $T = T(\theta)$ defined previously, there exists a solution θ that solves equation (27). By construction of $T = T(\theta)$, we know that $(1-\beta)(1+\bar{r}(\theta,\ \cdot) - T(\theta)) + T(\theta) > 0$. In addition, using the natural restriction $\eta \in [-(1-\delta_h), \infty]$ together with $T(\theta) < 0$ ensures $1 + \bar{r}(\theta,\ \cdot) - T + (1-\theta)\eta > 0$. Thus, the Inada conditions imply

$$\lim_{\theta \nearrow 1} \Omega_\theta(\theta, T(\theta)) = -\infty$$

$$\lim_{\theta \searrow 0} \Omega_\theta(\theta, T(\theta)) = +\infty$$

Therefore, given $T = T(\theta)$, we know that there exists a $\theta \in [0, 1]$ that solves (27), establishing the existence of an equilibrium $(\theta^*,\ T^*)$ with $T < 0$.

Finally, it is straightforward that the *Principle of Optimality* guarantees the uniqueness of the equilibrium.

Step 4: proof of part (iv.)

We now show that it is always optimal to subsidize the risky asset, $\tau_h < 0$. Strict concavity of $\frac{\eta}{1+\bar{r}(\theta^*,\ \cdot)-T^*+(1-\theta*)\eta}$ in η and Jensen's inequality imply

$$\mathbb{E}\left[\frac{\eta}{1+\bar{r}(\theta^*,\ \cdot) - T^* + (1-\theta^*)\eta}\right] < \frac{\mathbb{E}[\eta]}{1+\bar{r}(\theta^*, \cdot) - T^* + (1-\theta^*)\mathbb{E}[\eta]} = 0$$

By the upper and lower bounds on T, we know that $(1-\beta)(1+\bar{r}(\theta^*,\ \cdot) - T^*) + T^* > 0$. Hence, equation (27) implies $(r_k(\theta^*) - \delta_k) - (r_h(\theta^*) - \delta_h) \leq 0$. Now, we focus on equation

(26): The second and third term can be rewritten as

$$((r_k(\theta^*) - \delta_k) - (r_h(\theta^*) - \delta_h)) \cdot \frac{\nu(1-\beta)(1+\bar{r}(\theta^*,\ \cdot)) - (1-\beta-2\beta\nu+\nu)T^*}{((1-\beta)(1+\bar{r}(\theta^*,\ \cdot) - T^*) + T^*)(1+\bar{r}(\theta^*,\ \cdot) - T^*)}$$

Knowing that $T^* \in [-\frac{1-\beta}{\beta}(1+\bar{r}(\theta^*,\ \cdot)), 0]$, it is straightforward to verify that $\nu(1-\beta)(1+\bar{r}(\theta^*,\ \cdot)) - (1-\beta-2\beta\nu+\nu)T^* > 0$. Therefore, the complete expression is negative such that equation (26) implies

$$\mathbb{E}\left[\frac{(r_k(\theta^*) - \delta_k) - (r_h(\theta^*) - \delta_h + \eta)}{1+\bar{r}(\theta^*,\ \cdot) - T^* + (1-\theta^*)\eta}\right] > 0 \tag{30}$$

The optimal income tax rates (τ_k, τ_h) provide the incentive such that in competitive equilibrium, it is optimal for each household to choose θ^*. Hence

$$\begin{aligned} 0 &= \mathbb{E}\left[\frac{((1-\tau_k)r_k(\theta^*) - \delta_k) - ((1-\tau_h)r_h(\theta^*) - \delta_h + \eta)}{1+\bar{r}(\theta^*,\ \cdot) - T^* + (1-\theta^*)\eta}\right] \\ &= \mathbb{E}\left[\frac{(r_k(\theta^*) - \delta_k) - (r_h(\theta^*) - \delta_h + \eta) + (\tau_h r_h(\theta^*) - T^*)\frac{1}{\theta}}{1+\bar{r}(\theta^*,\ \cdot) - T^* + (1-\theta^*)\eta}\right] \end{aligned}$$

Solving for τ_h yields

$$\tau_h = -\frac{\mathbb{E}\left[\frac{\theta^*((r_k(\theta^*)-\delta_k)-(r_h(\theta^*)-\delta_h+\eta))-T^*}{1+\bar{r}(\theta^*,\ \cdot)-T^*+(1-\theta^*)\eta}\right]}{\mathbb{E}\left[\frac{r_h(\theta^*)}{1+\bar{r}(\theta^*,\ \cdot)-T^*+(1-\theta^*)\eta}\right]}$$

By (30) and $T^* < 0$, we conclude that the numerator is positive. Multiplying with (-1) and dividing by a positive number, we arrive at $\tau_h < 0$, which finally proofs part (iv.) of the proposition.