Rational expectations equilibrium and the strategic choice of costly information

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Abstract

This paper studies costly information acquisition in one-good production economies when agents acquire private information and prices transmit information. Before asset markets open, agents choose the quality of their private information. After this information stage, agents trade assets in sequentially complete markets taking into account their private information and the information revealed by equilibrium prices (rational expectations equilibrium, (Radner, R., 1979. Rational expectations equilibrium: generic existence and the information revealed by prices, Econometrica 47, 655–678.)). An overall equilibrium in asset and information market is defined as a Nash equilibrium of the information game in which agents’ actions are information choices and their utility payoffs are the ex-ante expected utilities of the corresponding rationale expectations equilibrium. This paper shows that for a generic set of economies parameterized by endowments and productivity shocks, an overall equilibrium in information and asset market (a Nash equilibrium of the induced information game) with costly information acquisition and fully-revealing prices exists. In other words, informational efficiency is in general consistent with costly information acquisition.

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1. Introduction

Motivated by Hayek’s insight (Hayek, 1945) that market prices perform an exceptional task in transmitting information, a large literature has studied the informational efficiency of competitive equilibria in economies with private information. For economies with finitely many states.
of private information, Radner (1979) showed that rational expectations equilibria (REE) are generically fully-revealing. In other words, for most economies equilibrium prices transmit all privately available information\(^1\). This full-revelation result has been criticized for at least two reasons. First, it appears to be refuted by the empirical evidence.\(^2\) Second, the complete and perfect transmission of private information by market prices seems to be inconsistent with costly information acquisition (Grossman and Stiglitz, 1980). After all, who would use scarce resources to gather information if the same information can be extracted from the movement of publicly observable market prices? This paper deals with the second type of criticism. More specifically, the Grossman-Stiglitz argument does not apply when agents take into account that without their own information acquisition activity, there would be less information contained in equilibrium prices.

The formal analysis confines attention to one-good production economies with sequentially complete asset markets, a finite number of (types of) agents, and a finite number of fundamental states. An information structure is defined as a partition of the finite set of fundamental states (“noiseless” information). Thus, there are finitely many information structures, and for each information structure there are finitely many information states, called signals. There are three periods. In the first period (ex-ante stage), agents decide on the quality of their private information by purchasing a particular partition of the finite state space. In addition, they trade assets with signal-contingent payoffs, one Arrow-security for each joint signal that is either directly or indirectly (through stock prices) observed by all agents. In the second period, agents trade stocks (make a capital allocation decision) and a complete set of assets with state-contingent payoffs taking into account their private information and the information revealed by equilibrium prices. Since in equilibrium there are at least as many signal-contingent Arrow-securities as there are joint signals revealed by stock prices, the asset structure is sequentially complete if stock prices are fully-revealing. In this case, there is a one-to-one relation between stock prices and payoffs of the signal-contingent securities, and these securities can therefore be interpreted as stock options (derivatives). In the final period, agents consume.

For given choices of information quality (information structures) by agents, this paper uses the standard solution concept of rational expectations equilibrium (REE, Radner, 1979). Information acquisition, however, is modeled as a game in which agents’ actions are information choices and agents’ utility payoffs are the expected utilities of the REE associated with the profile of information choices. This paper shows that for a generic set of economies parameterized by endowments and productivity shocks, an overall equilibrium in asset and information market (a Nash equilibrium of the information game) with costly information acquisition and fully-revealing stock prices exists. Thus, informational efficiency is in general consistent with costly information acquisition.

The existence of an overall equilibrium with costly information acquisition and fully-revealing prices is proved as follows. First, it is shown that for given information choices by agents and for a generic set of economies parameterized by endowments and productivity parameters, a fully-revealing REE exists if at least one agent has made a non-trivial information choice (has acquired some information). This part of the argument heavily relies on Radner’s original insight (Radner, 1979) that any fully-revealing full-communication (pooled-information) equilibrium is

\(^1\) For work on partially-revealing REE without the ad-hoc introduction of noise traders, see, for example, Ausubel (1990), Polemarchakis and Siconolfi (1993), and Pietra and Siconolfi (1997).

\(^2\) For a recent survey of the empirical literature see, for example, Campbell et al. (1997).
also a fully-revealing REE.\(^3\) Thus, there is a well-defined information game in which any profile of information choices (actions) with a non-trivial information choice by at least one agent gives rise to a fully-revealing REE, and in which no information acquisition by all agents leads to a non-informative REE (which always exists). The existence of a Nash equilibrium of this information game (an overall equilibrium) immediately follows from standard game-theoretic results since the number of information choices is finite.\(^4\) Finally, it is shown that if the cost of acquiring some information is small enough, then the non-informative REE is not a Nash equilibrium because at least one agent has an incentive to deviate, that is, at least one agent will gain from moving the economy to a fully-revealing REE. This last result follows from the following two observations. First, because asset markets are sequentially complete the equilibrium allocations are ex-ante Pareto efficient relative to the chosen information structure, and moving from a non-informative REE to any fully-revealing REE therefore does not decrease social welfare. Second, in production economies information has social value in the sense that an increase in information strictly increases social welfare, which immediately implies that at least one agent must strictly gain from the increase in information that is associated with moving from the non-informative REE to any fully-revealing REE.

2. Literature

Most of the previous literature on endogenous information acquisition in rational expectation models has relied on parametric examples of exchange economies.\(^5\) In exchange economies with risk averse agents and common priors, however, information has no social value in the sense that the ex-ante Pareto efficient consumption allocations are signal-independent (Dreze, 1960; Hirshleifer, 1971).\(^6\) Thus, the existence argument provided in this paper breaks down. Indeed, for the parametric examples considered in the literature, no overall equilibrium with costly information acquisition and fully-revealing prices exists, even if agents choose information strategically.\(^7\) In contrast, in production economies information has social value even if the common prior assumption holds. Marshall (1974) provides the first general analysis of the social value of information in production economies, but he does not provide rigorous proofs of the main propositions. In this paper, we formally show that when we parameterize production economies by endowments and productivity shocks, then generically information has social value. As argued above, this implies that an overall equilibrium with costly information acquisition and fully-revealing prices

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\(^3\) Radner’s original result (Radner, 1979) does not cover the case considered here because he considers an exchange economy with no signal-contingent assets and perturbs the prior beliefs of agents (therefore possibly ruling out economies with common priors).

\(^4\) The argument shows the existence of a Nash equilibrium, but not the existence of a pure-strategy Nash equilibrium. However, Harsanyi’s (1973) argument can be directly applied to the current framework demonstrating the existence of a pure-strategy Nash equilibrium for economies arbitrarily close (in the utility payoff space) to the original economy. Notice also that for each information structure there can be multiple equilibria, although generically there are only finitely many. Hence, the information game is only defined with respect to a particular selection criterion. However, the result that no information acquisition is not an equilibrium outcome holds regardless of which selection criterion is chosen.

\(^5\) See, for example, Grossman and Stiglitz (1980), Hellwig (1980), and Verrecchia (1982). Laffont (1985) provides a welfare analysis for this parametric class of economies.

\(^6\) See, for example, theorem 1 in Schlee (2001) for a general proof.

\(^7\) The previous literature has analyzed economies with a continuum of agent, but this result also holds for a finite number of agents. This can easily be seen from Hellwig’s proposition 4.3 (Hellwig, 1980), which deals with the case of a finite number of agents and no noise.
exists if the choice of information is modeled as the strategic interaction of a finite number (of types) of agents. It remains an open question whether an overall equilibrium with fully-revealing prices exists when the information market is competitive.

An important difference between this paper and the existing literature on endogenous information acquisition in rational expectation models is the combination of competitive behavior in asset markets with strategic behavior in the market for information. In other words, agents take into account the effect of their behavior on equilibrium prices when they choose the quality of information, but not when they trade in asset markets. The hybrid assumption that “the same” agent behaves myopically in one market and strategically in another market has a long tradition in international trade theory (Norman and Dixit, 1979), and has also been used in several papers on financial innovation (Allen and Gale, 1995). In the present context of REE, it seems a reasonable assumption in at least two types of applications. First, consider the case in which a few major investment banks decide on the overall amount of research spending over the next 5–10 years (strategic choice of information quality), and each investment bank employs a large number of traders (asset trading in competitive markets). As a second application, consider a situation in which governments (central banks) of individual countries choose the accuracy and timeliness of the macroeconomic data they release (choice of fiscal and monetary transparency by countries), and private investors take into account the news releases of individual governments (central banks) when trading assets in international financial markets. In both these applications the hybrid assumption of competitive behavior in asset markets and strategic behavior in the market for information seems to capture important aspects of the economic problem.

Finally, it seems worth pointing out that in contrast to the present paper, most of the literature on rational expectations models does not allow for the possibility of trade of signal-contingent securities before the arrival of information. However, this type of market structure is not implausible since agents have an incentive to open up security (insurance) markets for any event that is publicly observed in equilibrium, and it has already been used in the important work by Marshall (1974). Moreover, Mas-Colell et al. (1995) suggest that this market structure provides a natural extension of Radner’s original framework, and they also note the equivalence between signal-contingent securities and options when stock prices are fully-revealing. The idea that option contracts can be used to complete the market structure when stock prices are fully-revealing goes back to Ross (1976).

3. Model

We consider a one-good production economy with three periods, \( t = 0, 1, 2 \). In period \( t = 0 \) (ex-ante), agents choose the quality of information and trade signal-contingent Arrow-
securities in competitive markets. In period $t=1$ (interim), agents receive information about the fundamental state, make a production decision (purchase equity contracts), and trade state-contingent Arrow-securities in competitive markets. In period $t=2$ (ex post), agents consume. To streamline the exposition, all definitions are stated only for the case of a full-communication (pooled-information) equilibrium (FCE), that is, we immediately focus on the case in which no informational asymmetries persist in equilibrium. Radner’s observation (Radner, 1979) that any fully-revealing FCE is also a fully-revealing REE then allows us to interpret any result that is proved for fully-revealing FCE as a statement about fully-revealing REE. The analysis begins with the definition of FCE for given information choices, and then proceeds to define an overall equilibrium in information and asset market by endogenizing the information choice.

3.1. The economy

There is a finite number of (types of) agents $i \in I \equiv \{1, \ldots, I\}$ and a finite number of possible fundamental states (states of nature) $s \in S \equiv \{1, \ldots, S\}$. Information is modeled as a partition of the set $S$. The partition chosen by agent $i$ (agent $i$’s action in the information game) is denoted by $a_i \equiv \{S(1), \ldots, S(\Sigma_i)\}$, where $S(\sigma_j) \subset S$ for all $\sigma_j = 1, \ldots, \Sigma_i$. Clearly, we have $\Sigma_i \leq S$. We denote the join (the coarsest common refinement) of the list of partitions $a_1, \ldots, a_I$ by $\alpha \equiv \{S(1), \ldots, S(\Sigma)\}$, where $S(\sigma) \subset S$, for all $\sigma = 1, \ldots, \Sigma$. This partition is the information structure that results if all agents pool their private information. Clearly, we have $\Sigma_i \leq \Sigma \leq S$. We call the number $\sigma_i$ (the set $S(\sigma_i) \subset S$) the signal received by agent $i$ and the number $\sigma$ (the set $S(\sigma) \subset S$) the joint signal received by all agents (recall that the focus is on FCE). Clearly, the partition approach to modeling information structures is equivalent to the approach in which each information structure is defined by a signal function $\tilde{\sigma}_i : S \rightarrow \mathbb{R}$. This follows immediately from the observation that each signal function induces a partition of $S$, namely the partition with elements $\{s \in S|\tilde{\sigma}(s) = c\}$ for any real number $c$ that is in the image set of $\tilde{\sigma}_i$.

There is a common (objective) prior probability that state $s \in S$ occurs, which we denote by $\pi(s) \in [0, 1]$. Notice that the formal results proved in this paper also hold when prior probabilities differ across agents. We denote the joint probability that signal $\sigma$ and state $s$ occur by $\pi(\sigma, s) \in [0, 1]$. Since $\pi(\sigma, s) = \pi(\sigma|s)\pi(s)$, we have $\pi(\sigma, s) = 0$ for all pairs $(\sigma, s)$ with $s \notin S(\sigma)$ and $\pi(\sigma|s) = \pi(s)$ for all pairs $(\sigma, s)$ with $s \in S(\sigma)$. For simplicity, we assume $\pi(s) > 0$ for all states $s \in S$.

A simple example might clarify the notation. Assume that there are two agents, $i \in I = \{1, 2\}$, and two fundamental states, $s \in S = \{1, 2\}$. There are two possible partitions of $S$, namely $\{\{1\}, \{2\}\}$ and $\{\{1, 2\}\}$. If both agents decide not to acquire information, $a_i = \{\{1, 2\}\}$ for all $i$, then the joint information structure is non-informative: $\alpha = \{\{1, 2\}\}$. In this case, we have only one private signal and one joint signal, $\Sigma_1 = \Sigma_2 = \Sigma = 1$, and joint probabilities are equal to the marginal probabilities: $\pi(1) = \pi(s)$. If at least one agent acquires information, $a_i = \{\{1\}, \{2\}\}$ for at least one $i$, then the joint information structure is fully informative, $a_i = \{\{1\}, \{2\}\}$, and we have two joint signal realizations, $\sigma = 1, 2$, with corresponding joint probabilities $\pi(1, 1) = \pi(1), \pi(2, 2) = \pi(2)$ and $\pi(1, 2) = \pi(2, 1) = 0$.

Agents receive an initial endowment of the good in period 1 and period 2. The first-period endowment of agent $i$, $\omega_{1i}$, is strictly positive and state-independent: $\omega_{1i} \in \mathbb{R}_{++}$. The second-period endowment, $\omega_{2i}$, is strictly positive and state-dependent: $\omega_{2i} \in \mathbb{R}^{\Sigma_i}_{++}$. The second-period endowment vector of the economy is $\omega_2 = (\omega_{21}, \omega_{22}, \ldots, \omega_{2I}) \in \mathbb{R}^{\Sigma_I}_{++} \equiv \Omega$.

There is a finite number of firms, $j \in J \equiv \{1, \ldots, J\}$, represented by linear production functions (constant-returns-to-scale with one input factor). Firm $j$ produces output $z_j(s)k_j(\sigma) \in \mathbb{R}_+$ in the
second period if it has invested capital \( k_j(\sigma) \in \mathbb{R}_+ \) in the first period, where \( z_j(s) \in \mathbb{R}_{++} \) is a strictly positive parameter measuring the productivity of firm \( j \) in state \( s \). We denoted by \( z_j = (z_j(1), \ldots, z_j(S)) \in \mathbb{R}_+^S \) the productivity vector of firm \( j \) and by \( z = (z_1, \ldots, z_J) \in \mathbb{Z} \equiv \mathbb{R}_+^JS \) the entire productivity array.

We parameterize the set of economies by second-period endowments and productivity shocks. A property holds generically if and only if it holds for a subset of \( \Omega \times Z \) with full Lebesgue measure.

Agent \( i \)'s preferences over consumption bundles, \( c_i \in \mathbb{R}_+^{\Sigma S} \), are of the von Neumann-Morgenstern type. In addition, preferences are assumed to be smooth (to apply differential topology), strictly monotone (to avoid zero prices for positive-probability events) with strictly concave Bernoulli utility function satisfying a boundary condition (to use first-order conditions). More specifically, we assume that the utility function representing preferences, \( U_i : \mathbb{R}_+^{\Sigma S} \rightarrow \mathbb{R} \), is of the form

\[
U_i(c_i) = \sum_{\sigma,s} \pi(\sigma, s) u_i(c_i(\sigma, s), s).
\]

Moreover, for each \( s \in S \) the function \( u_i(\cdot, s) : \mathbb{R}_+ \rightarrow \mathbb{R} \) is smooth and satisfies: \( u'_i(x, s) > 0 \), for all \( x \in \mathbb{R}_+, u''_i(x, s) < 0 \) for all \( x \in \mathbb{R}_+ \), and \( \lim_{x \to 0} u'_i(x, s) = +\infty \).

The assumption of smooth preferences (and smooth production functions) is stronger than needed: \( u_i \) twice continuously differentiable suffices if in the proposition ‘smooth manifold’ is replaced by ‘\( \mathcal{C}^1 \)-manifold’. The smoothness convention, however, is standard (Debreu, 1972) and will be adopted here. Evidently, the assumptions made so far imply that \( U_i : \mathbb{R}_+^{\Sigma S} \rightarrow \mathbb{R} \) is smooth, strictly monotone, strictly concave, and satisfies a corresponding boundary condition.

### 3.2. Equilibrium

As mentioned before, the formal analysis focusses on FCE. Thus, at time 1 all agents observe the joint signal \( \sigma \in \{1, \ldots, \Sigma\} \), which means that at time \( t=0 \) agents trade \( \Sigma \) signal-contingent Arrow-securities in competitive markets, one Arrow-security for each joint signal \( \sigma \). Arrow-security \( \sigma \) pays off one unit of the good in period \( t=1 \) if signal \( \sigma \) has been received, and nothing otherwise. We denote the quantity of security \( \sigma \) purchased (sold if negative) by agent \( i \) by \( \theta_{0i} \) and the vector of asset trades by \( \theta_{0i} \in \mathbb{R}_+^{\Sigma J} \). Notice that we imposed no restrictions on the short-sale of assets. The price of security \( \sigma \) is denoted by \( q_{0\sigma} \) and the entire asset price vector by \( q_0 \in \mathbb{R}_+^{\Sigma J} \).

At time \( t=1 \), agents decide how much to invest in their individual production technologies and trade \( S \) state-dependent Arrow-securities in competitive markets. Agent \( i \) participates in the production process of firm \( j \) by purchasing \( \theta_{ei}(\sigma) \in \mathbb{R}_+ \) shares of this stock company. An agent who has purchased \( \theta_{eij}(\sigma) \) shares of company \( j \) at price \( q_{eij}(\sigma) \in \mathbb{R}_+ \) receives dividend payments \( z_j(s)k_j(\sigma)\theta_{eij}(\sigma) \) if \( (\sigma, s) \) occurs. We denote the \( \sigma \)-dependent equity-trading vector of agent \( i \) by \( \theta_{ei}(\sigma) \in \mathbb{R}_+^{\Sigma J} \) and the entire equity trading plan by \( \theta_{ei} \in \mathbb{R}_+^{\Sigma J} \). Further, the \( \sigma \)-dependent equity price vector is denoted by \( q_e(\sigma) \in \mathbb{R}_+^{\Sigma J} \) and the entire equity price function by \( q_e \in \mathbb{R}_+^{\Sigma J} \). Arrow-security \( s \) pays off one unit of the good in period \( t=2 \) if state \( s \) occurs, and nothing otherwise. We denote the quantity of security \( s \) purchased (sold if negative) by agent \( i \) if information \( \sigma \) has been received by \( \theta_{1i}(\sigma) \), the \( \sigma \)-dependent security trade vector by \( \theta_{1i}(\sigma) \in \mathbb{R}_+^S \), and the entire security trading plan by \( \theta_{1i} \in \mathbb{R}_+^{\Sigma S} \). Notice that we again did not impose any restrictions on the short-sale of Arrow-securities. The price of security \( s \) if \( \sigma \) is denoted by \( q_{1s}(\sigma) \), the \( \sigma \)-dependent price vector
by \( q_1(\sigma) \in \mathbb{R}_+^S \), and the entire security price function by \( q_1 \in \mathbb{R}^{\Sigma S}_+ \). Finally, we use the notation \( q=(q_0,q_1,q_e) \) and \( \theta=(\theta_0,\theta_1,\theta_e) \).

For notational simplicity, we assume that firms have no initial capital stock and finance all investment through the issue of new shares (initial public offering). We also normalize the number of outstanding shares to one, which implies that in equilibrium the capital stock of firm \( j \) is equal to its stock price: \( k_j(\sigma) = q_e(\sigma) \).

The sequential budget constraint of agent \( i \) reads:

\[
\sum_{\sigma} q_{0\sigma} \theta_{0i\sigma} = 0 \\
\forall \sigma : \sum_s q_{1s}(\sigma) \theta_{1is}(\sigma) + \sum_j q_{ej}(\sigma) \theta_{eij}(\sigma) = \theta_{0i\sigma} + \omega_{1i} \tag{2.1}
\]

\[
\forall \sigma, s : c_i(\sigma, s) = \omega_{2i}(s) + \theta_{1is}(\sigma) + \sum_j z_i(s) k_j(\sigma) \theta_{eij}(\sigma)
\]

The budget set is defined as

\[
B_i(q) := \left\{ (c_i, \theta_i) \in \mathbb{R}_+^{\Sigma S} \times \mathbb{R}^{\Sigma(1+\Sigma+S+J)} | (c_i, \theta_i) \text{satisfy (2.1)} \right\}.
\]

Of course, the budget set not only depends on prices, but also on the information structure, \( a \), and the endowment and productivity parameters, \( (\omega_2,z) \). Notice that the period-0 budget constraint assumes that no resources are used for information acquisition activity. Thus, the cost of information acquisition introduced below should be interpreted as a utility cost due to the disutility from the time spent acquiring information. This assumption is only made to simplify the exposition.

Finally, the market clearing conditions read:

\[
\sum_i \theta_{0i} = 0 \\
\forall \sigma : \sum_i \theta_{1i}(\sigma) = 0, \quad \sum_i \theta_{ei}(\sigma) = 1, \quad \sum_j k_j(\sigma) = \sum_i \omega_{1i} \tag{2.2}
\]

\[
\forall \sigma, s : \sum_i c_i(\sigma, s) = \sum_i \omega_{2i}(s) + \sum_j z_i(s) k_j(\sigma)
\]

Note that asset market clearing in conjunction with the individual budget constraint implies goods market clearing (Walras’ law). In the subsequent analysis, we therefore omit the goods market clearing conditions.

Notice that we have not introduced additional measurability constraints when defining the budget set of agent \( i \), that is, each agent’s trading plan may depend on \( \sigma \), which is the signal corresponding to the join of all individual partitions. In a similar vein, we have introduced one signal-contingent Arrow-security for each signal \( \sigma \), which implicitly assumes that in equilibrium \( \sigma \) is known to all agents. As mentioned before, this assumption is justified if agents pool their private information. This idea of direct communication among agents gives rise to the concept of a full-communication equilibrium (FCE, Radner, 1979). In the following, denote the non-informative partition \( a_i \equiv \{S\} \) by \( \bar{a} \). Clearly, if all agents choose to be uninformed, \( (a_1, \ldots, a_I) = (\bar{a}, \ldots, \bar{a}) \), then the resulting information structure is \( \bar{a} \). We have the following definition:

**Definition 1.** For given information choice \( (a_1, \ldots, a_I) \), a full-communication equilibrium (FCE) is an asset price vector \( q \) and a vector of consumption and trading plans \( (c,\theta) \) such that:
(i) Agents maximize: for given \( q \), \((c_i, \theta_i)\) solves
\[
\max_{(c_i, \theta_i) \in B_i(q)} U_i(c_i).
\]

(ii) Markets clear: \((c, \theta)\) satisfies (2.2).

For any \((a_1, \ldots, a_I) \neq (\bar{a}, \ldots, \bar{a})\), a fully-revealing FCE is a FCE with

(iii) Stock prices are fully-revealing:
\[
\forall \sigma \neq \sigma' \exists j : q_{e_j}(\sigma) \neq q_{e_j}(\sigma').
\]

In our definition of equilibrium we do not mention the capital allocation since the capital stock of firm \( j \) is determined by its stock price and the number of outstanding shares, which is equalized to one: \( k_j(\sigma) = q_{e_j}(\sigma) \).

So far, we have only discussed a sequential market structure. However, for any economy and information structure, we can also define a corresponding contingent-market equilibrium with only one round of asset trade. Clearly, a contingent market equilibrium exists. It is straightforward to show that the allocation of any contingent-market equilibrium is also the allocation of a FCE as defined above. Since for a given allocation we can always back out prices, a FCE exists.

The concept of FCE certainly does not capture the idea of transmission of private information by prices. In a REE (Radner, 1979), we therefore require that each agent can only use his own information and the information contained in prices. Depending on the amount of information revealed by equilibrium prices, this requirement adds measurability constraints. However, if prices are fully-revealing, that is, if the price function is invertible, then no constraints have to be added. Put differently, any fully-revealing FCE is also a fully-revealing REE. The following summarizes the preceding discussion.

3.2.1. Facts

(i) For any information structure and economy, a FCE exists.

(ii) Any fully-revealing FCE is a fully-revealing REE.

Before asset markets open and capital allocation decisions are made, agents choose the quality of their private information, that is, they choose \( a_i \in A \), where \( A \) stands for the set of all partitions of \( S \). For any \((a_1, \ldots, a_I) \neq (\bar{a}, \ldots, \bar{a})\), denote by \( \text{FCE}_f(a_1, \ldots, a_I) \) the set of fully-revealing FCE (REE) associated with \((a_1, \ldots, a_I) \neq (\bar{a}, \ldots, \bar{a})\). Clearly, this set is the same for all partition profiles, \((a_1, \ldots, a_I)\), that have the same join, \( a \), but to emphasize the dependence of this set on the action of individual agents, we continue to use the notation \( \text{FCE}_f(a_1, \ldots, a_I) \). Below we will show that for a generic set of economies, this set is non-empty for all \((a_1, \ldots, a_I) \neq (\bar{a}, \ldots, \bar{a})\). Hence, for each action profile \((a_1, \ldots, a_I) \neq (\bar{a}, \ldots, \bar{a})\) we can (again generically) define a utility payoff, \( \tilde{V}_i = \tilde{V}_i(a_1, \ldots, a_I) \), for each agent, namely the ex-ante expected utility of agent \( i \) in one of the fully-revealing FCE associated with the join of \((a_1, \ldots, a_I)\). For \((a_1, \ldots, a_I) = (\bar{a}, \ldots, \bar{a})\), we use the ex-ante expected utility associated with one of the non-informative FCE. Let \( \varphi_i(a_i) \in \mathbb{R}_+ \) the utility cost agent \( i \) incurs when acquiring the information structure \( a_i \). As mentioned before, the assumption that information acquisition has only utility cost (time spend acquiring information), but no direct resource cost, is made to streamline the analysis. For each agent \( i \), we can then define
a preference relation over \((A)^I\) by letting \(V(a_1, \ldots, a_I) = \bar{V}(a_1, \ldots, a_I) - \varphi_i(a_i)\) be the utility of agent \(i\) if the outcome is \((a_i, \ldots, a_I)\). Thus, we have defined a game \(G = (I, (A)^I, (V_1, \ldots, V_I))\), which we refer to as the information game. We assume \(\varphi_i(\bar{a}) = 0\) and \(\varphi_i(a_i) > 0\) for all \(a_i \neq \bar{a}\). That is, any information acquisition has strictly positive cost and no information acquisition has zero cost. Notice that we assume that the action space, \(A\), is the same for all agents. However, any difference in agents’ ability to acquire information can be introduced through differences in the cost functions \(\varphi_i\).

Although we show below that for a given information structure the set of fully-revealing REE is non-empty and finite, we have no proof of uniqueness. Hence, when we define the information game, we need to employ an arbitrary convention (selection criterion). However, the results shown below hold regardless of the particular selection criterion used to define the information game.

Having defined the information game \(G\), we are now in a position to define an overall equilibrium.

**Definition 2.** An overall equilibrium in asset and information markets is a Nash equilibrium of the information game \(G\).

4. Equilibrium

Given the assumptions made so far, first-order condition are necessary and sufficient conditions for utility maximization. Hence, for given information choice \((a_1, \ldots, a_I)\), a vector \((c, \theta, q) \in \mathbb{R}^{\Sigma S} \times \mathbb{R}^{\Sigma(1+S+J)} \times \mathbb{R}^{\Sigma(1+S+J)}\) is a FCE if and only if it satisfies the budget constraints

\[
\forall i : \sum_{\sigma} q_{0\sigma} \theta_{0i\sigma} = 0
\]

\[
\forall i, \sigma : \sum_s q_{1s}(\sigma) \theta_{1is}(\sigma) + \sum_j q_{ej}(\sigma) \theta_{eij}(\sigma) = \theta_{0i\sigma} + \omega_{1i}
\]

\[
\forall i, \sigma, s : c_i(\sigma, s) = \omega_{2s}(s) + \theta_{1is}(\sigma) + \sum_j z_j(s) q_{ej}(\sigma) \theta_{eij}(\sigma)
\]

and asset market clearing conditions (goods market clearing is satisfied by Walras’ law)

\[
\sum_i \theta_{0i} = 0, \quad \forall \sigma : \sum_i \theta_{1i}(\sigma) = 0, \quad \forall \sigma : \sum_i \theta_{ei}(\sigma) = 1,
\]

and there exists for each agent \(i\) a vector of Lagrange multipliers \((\lambda_{0i}, \lambda_{1i}) \in \mathbb{R}^{\Sigma+1}\) such that

\[
\forall i, \sigma, s : \pi(\sigma, s) u'_i(c_i(\sigma, s), s) = \lambda_{1i}(\sigma) q_{1s}(\sigma)
\]

\[
\forall i, j, \sigma : \sum_s \pi(\sigma, s) z_j(s) u'_i(c_i(\sigma, s), s) = \lambda_{1j}(\sigma)
\]

\[
\forall i, \sigma : \lambda_{1i}(\sigma) = \lambda_{0i} q_{0\sigma}
\]

Notice that in (3.1) and (3.3) we have already used the equilibrium condition \(k_j(\sigma) = q_{ej}(\sigma)\) and that asset market clearing implies goods market clearing.
Rearranging the first two Eqs. in (3.3) and using the third to eliminate $\lambda_{1t}(\sigma)$, we can replace (3.3) by

$$\forall i, \sigma, s : \frac{1}{\lambda_{0i}} \pi(\sigma, s)u_i'(c_i(\sigma, s), s) = q_0 \sigma q_{1s}(\sigma)$$
$$\forall j, \sigma : \sum_s q_{1s}(\sigma)z_j(s) = q_{ej}(\sigma).$$

(3.4)

Hence, we can identify a FCE with a vector $(c, \theta, \lambda, q) \in \mathbb{R}^{\Sigma S} \times \mathbb{R}^{\Sigma(1+S+J)} \times \mathbb{R}_+ \times \mathbb{R}^{\Sigma(1+S+J)}$ solving (3.1), (3.2), and (3.4). Moreover, a fully-revealing FCE (REE) can be identified with a vector $(c, \theta, \lambda, q)$ solving (3.1), (3.2), (3.4), and the additional inequalities

$$\forall \sigma \neq \sigma \exists j : q_{ej}(\sigma) \neq q_{ej}(\sigma').$$

(3.5)

Our definition of equilibrium has several (irrelevant) indeterminacies built in. First, for all $(\sigma, s)$ with $s \notin S(\sigma)$ (and therefore $\pi(\sigma, s) = 0$), we must have $q_{1s}(\sigma) = 0$, which implies that a continuum of security choices $\theta_{1is}(\sigma)$ are optimal. We set $\theta_{1is}(\sigma) = 0$ in this case. Notice that the first $I \Sigma S$ first-order conditions in (3.4) have now been reduced to only $I S$ first-order conditions that are not trivially satisfied. Second, there is a nominal security-price indeterminacy, which we eliminate by setting $q_0 \Sigma = 1$ and $q_{1s}(\sigma) = 1$ if $(\sigma)$ is the largest element of $S(\sigma)$. For the remaining asset prices we confine attention to strictly positive prices. We denote the set of asset price vectors, $q$, satisfying all the equality and inequality restrictions by $Q \subset \mathbb{R}^{\Sigma(1+S+J)}$. Clearly, $Q$ is a smooth manifold of dimension $S - 1 + \Sigma J$. Finally, in period $t = 1$ agents trade $S$ Arrow-securities and $J$ stocks, but there are only $S$ fundamental states of uncertainty. In equilibrium, there will be a unique set of asset prices (once the above mentioned normalization is introduced) that do not permit arbitrage opportunities, but given these arbitrage-free prices agents can achieve the same consumption vector with a continuum of different portfolio choices. We resolve this indeterminacy by setting $\theta_{ej}(\sigma) = (1/\sigma)$ so that in equilibrium each agent is an equal shareholder of each stock company regardless of the signal realization. We denote the set of trading plans, $\theta$, satisfying all the above equality restrictions by $\Theta \subset \mathbb{R}^{\Sigma(1+S+J)}$. Clearly, $\Theta$ is a smooth manifold of dimension $I S$.

Using the conventions discussed above, we introduce the set of all FCE, the set of all fully-revealing FCE, and the set of partially-revealing FCE for a given information structure $(a_1, \ldots, a_t)$ and economy $(\omega, z)$:

$$\text{FCE}(\omega, z) \equiv \left\{ (c, \theta, \lambda, q) \in \mathbb{R}^{\Sigma S} \times \Theta \times \mathbb{R}_{++} \times Q | (c, \theta, \lambda, q) \text{solves (3.1), (3.2), and (3.4)} \right\}$$

$$\text{FCE}_f(\omega, z) \equiv \left\{ (c, \theta, \lambda, q) \in \mathbb{R}^{\Sigma S} \times \Theta \times \mathbb{R}_{++} \times Q | (c, \theta, \lambda, q) \text{solves (3.1), (3.2), (3.4), and (3.5))} \right\}$$

$$\text{FCE}_p(\omega, z) \equiv \text{FCE}(\omega, z) \setminus \text{FCE}_f(\omega, z).$$

Clearly, these sets also depend on the information choices $(a_1, \ldots, a_t)$, but this dependency will be suppressed for the time being. Notice that the set of fully-revealing FCE is identical with the set of fully-revealing REE, $\text{FCE}_f(\omega, z) \equiv \text{REE}_f(\omega, z)$, but the set of partially-revealing
REE is in general a strict subset of the set of partially-revealing FCE: \( \text{REF}_p(\omega_2, z) \subset \text{FCE}_p(\omega_2, z) \). This observation in conjunction with lemma 2 below imply that generically there is no overall equilibrium in asset and information market with partially revealing prices.

By moving along different economies (but still fixing the information structure), we define the following equilibrium sets:

\[
E_f \doteq \left\{ (c, \theta, \lambda_0, q, \omega_2, z) \in \mathbb{R}^{\Sigma SI} \times \Theta \times \mathbb{R}^I_+ \times Q \times \Omega \times Z | (c, \theta, \lambda_0, q) \in \text{FCE}_f(\omega_2, z) \right\}
\]

\[
E_p \doteq \left\{ (c, \theta, \lambda_0, q, \omega_2, z) \in \mathbb{R}^{\Sigma SI} \times \Theta \times \mathbb{R}^I_+ \times Q \times \Omega \times Z | (c, \theta, \lambda_0, q) \in \text{FCE}_p(\omega_2, z) \right\}
\]

Following Balasko (1988), we will study the structure of the equilibrium sets \( \text{FCE}_f(\omega_2, z) \) and \( \text{FCE}_p(\omega_2, z) \) for generic \( (\omega_2, z) \) by first studying the sets \( E_f \) and \( E_p \). The latter sets are determined as the solution to finite-dimensional equation systems consisting of equalities and inequalities and exhibit the following manifold structure:

**Lemma 1.** For any information choice \((a_1, \ldots, a_I) \neq (\bar{a}, \ldots, \bar{a})\), the set \( E_f \) is a smooth manifold of dimension \( SI + SJ \) (the dimension of \( \Omega \times Z \)) and the set \( E_p \) is the union of \( \Sigma(\Sigma - 1)/2 \) smooth manifolds of dimension \( SI + SJ - J \).

**Proof.** See Appendix A. \( \square \)

Lemma 1 in conjunction with Sard’s theorem imply that generically equilibria are fully-revealing:

**Lemma 2.** For any information choice \((a_1, \ldots, a_I) \neq (\bar{a}, \ldots, \bar{a})\), there is a generic set of economies \( \Omega^* = Z^* \subset \Omega \times Z \) such that for all \((\omega_2, z) \in \Omega^* \times Z^*\) the set \( \text{FCE}_p(\omega_2, z) \) is empty and the set \( \text{FCE}_f(\omega_2, z) \) is non-empty and finite.

**Proof.** See Appendix A. \( \square \)

**Remark 1.** Since \( \text{REE}_p(\omega_2, z) \subset \text{FCE}_p(\omega_2, z) \), lemma 2 also shows that generically no partially-revealing REE exists for all information choices \((a_1, \ldots, a_I)\). Thus, generically there is no overall equilibrium in asset and information markets with partially revealing prices.

Lemma 1 and Sard’s theorem also imply that generically information has social value:

**Lemma 3.** For any information choice \((a_1, \ldots, a_I) \neq (\bar{a}, \ldots, \bar{a})\), there is a generic set of economies \( \Omega^* = Z^* \subset \Omega \times Z \) such that for all \((\omega_2, z) \in \Omega^* \times Z^*\) the set of fully-revealing FCE with signal-independent consumption allocation, \( c_i(\sigma, s) = c_i(\sigma', s) \forall i, \sigma, \sigma', s \), is empty.

The next proposition states our main result: overall equilibrium in asset and information market with fully-revealing asset prices is a well-defined concept and for small enough (but still positive) cost of information acquisition, the overall equilibrium always entails some degree of information acquisition.

**Proposition.** There is a generic set of economies \( \Omega^* \times Z^* \subset \Omega \times Z \), such that the information game, \( G = (I, A^I, (V_i)_{i=1}^I) \), is well-defined and has a Nash equilibrium, \((a_1^*, \ldots, a_I^*) \in A^I\). Moreover, there are cost functions, \( \varphi_1, \ldots, \varphi_I \), with \( \varphi_i(\bar{a}) = 0 \) for all \( i \) and \( \varphi_i(a_i) > 0 \) for all \( i \) and all \( a_i \neq \bar{a} \) so that \((a_1^*, \ldots, a_I^*) \neq (\bar{a}, \ldots, \bar{a})\).

**Proof.** Lemma 2 states the existence of a generic set of economies \( \Omega^* \times Z^* \subset \Omega \times Z \) so that for any economy \((\omega_2, z) \in \Omega \times Z\) each action profile \((a_1^*, \ldots, a_I^*) \neq (\bar{a}, \ldots, \bar{a})\) is associated with a
finite number of fully-revealing REE. Moreover, there always exists a non-informative REE and, also by lemma 2, there are only finitely many non-informative REE (for a generic set of economies). Invoking an arbitrary selection criterion, we can therefore define the information game \( G \) with derived utility payoff functions \( V_i = V_i(a_1, \ldots, a_I) = \tilde{V}_i(a_1, \ldots, a_I) - \varphi_i(a_i) \). Since the action space, \( (A)^I \), is finite, the existence of a Nash equilibrium, \((a_1^*, \ldots, a_I^*)\), follows immediately. Of course, this Nash equilibrium could be a mixed-strategy Nash equilibrium.

It is left to show that there are cost functions \( \varphi_1, \ldots, \varphi_I \) so that \((a_1^*, \ldots, a_I^*) \neq (\bar{a}, \ldots, \bar{a})\), that is, we need to show that there are cost functions \( \varphi_1, \ldots, \varphi_I \) so that at least one agent has an incentive to deviate form the action profile \((\bar{a}, \ldots, \bar{a})\). To see this, consider the action profile \((a_i^*, \tilde{a}_{-i})\), which is the action profile with \( a_i = \bar{a} \) for all \( i \neq \hat{i} \) and \( a_i \neq \bar{a} \). The claim is proved if

\[
\tilde{V}_i(a_i^*, \tilde{a}_{-i}) > \tilde{V}_i(\bar{a}, \ldots, \bar{a}),
\]

because (3.6) immediately implies the existence of a number \( \varphi_i(a_i) > 0 \) so that

\[
V_i(a_i^*, \tilde{a}_{-i}) > V_i(\bar{a}, \ldots, \bar{a}).
\]

Thus, it is left to show that (3.6) holds.

Let \( \text{REE}(\omega_2, z; a_i, \bar{a}_{-i}) \) be the set of fully-revealing REF associated with the action profile \((a_i, \bar{a}_{-i})\) and \( \text{REE}(\omega_2, z; \bar{a}, \ldots, \bar{a}) \) be the set of REE associated with the action profile \((\bar{a}, \ldots, \bar{a})\). Pick an arbitrary \((c, \theta, q) \in \text{REE}(\omega_2, z; a_i, \bar{a}_{-i})\) and an arbitrary \((\tilde{c}, \tilde{\theta}, \tilde{q}) \in \text{REE}(\omega_2, z; \bar{a}_1, \ldots, \bar{a}_I)\), where the consumption allocation \( \tilde{c} \) satisfies \( \tilde{c}_i(\sigma, s) = \bar{c}_i(\sigma', s) \) for all \( i, \sigma, \sigma' \), \( s \). The consumption allocations \( c \) is not only an equilibrium allocations, but also the solution to a corresponding social planner problem. Lemma 3 implies that \( c \neq \tilde{c} \), which in turn implies that social welfare associated with \( c \) must be strictly larger than the social welfare associated with \( \tilde{c} \). Thus, the ex-ante expected utility of at least one agent \( \hat{i} \) must be larger in the REE associated with the action profile \((a_i^*, \bar{a}_{-i})\):

\[
\tilde{V}_i(a_i^* \tilde{a}_{-i}) > \tilde{V}_i(\bar{a}, \ldots, \bar{a}).
\]

Since the action profiles \((a_1^*, \bar{a}_{-i})\) and \((a_i^*, \bar{a}_{-i})\) lead to the same joint information structure and therefore to the same fully-revealing REE if \( a_i = a_i^* \) (it does not matter which agent acquires the information), we have \( \tilde{V}_i(a_i^* \tilde{a}_{-i}) = \tilde{V}_i(a_i^* \tilde{a}_{-i}) \) if \( a_i = a_i^* \). Thus, (3.8) implies (3.6), which completes the proof. Notice that this proof also shows that the proposition holds regardless of the selection criterion used to define the game \( G \).

**Remark 2.** The proof of the proposition requires that “a small amount of information” is not too costly, that is, for at least one partition \( a \neq \bar{a} \) the cost \( \varphi_i(a) \) is arbitrarily small for all agents \( i = 1, \ldots, I \). If one is willing to assume that even the cost of acquiring full information, \( a_i = \{S\} \), is arbitrarily small for all agents, then any Nash equilibrium of the information game has \( a_i^* = \{S\} \) for at least one agent \( \hat{i} \), which implies \( a = \{S\} \). That is, in any equilibrium all agents are fully informed about the fundamental state at the interim stage.

5. Conclusion

This paper suggested a new equilibrium concept for private information economies with publicly observable prices and costly information acquisition by combining the assumption of strategic behavior in the information market (game theory) with the assumption of competitive behavior in the asset market (general equilibrium theory). The main result was the proof of existence of an equilibrium with costly information acquisition and fully-revealing prices.
There are at least two extensions of the present framework which promise to yield important insights into the functioning of private information economies with publicly observable prices and costly information acquisition. First, a more general framework that allows for an uncountable set of information choices would be of great interest from a theoretical and applied point of view. If the information structure is “noisy”, an uncountable set of information choices already arises even if the set of fundamental states is finite. Second, and somewhat related to the first point, introducing some notion of perfect competition into the information market is an important topic for future research. This would permit addressing the following issue: is full-revelation of private information by market prices consistent with costly information acquisition if both information and asset markets are perfectly competitive?

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Appendix A. Proof of Lemma 1

We begin with the set $E_f$.

(i) Non-emptiness: obvious.
(ii) Manifold structure: we need to show that for each point $e^* = (c^*, \theta^*, \lambda^*_{0i}, q^*, \omega^*_2, z^*) \in E_f$ there is a local parameterization with $SI + SJ$ free parameters. To this end, it is convenient to introduce notation decomposing all variables into independent variables (bar variables) and dependent variables (hat variables).

For the first $i = 1, \ldots, I - 1$ agents, we will use the consumption vector and some components of the security portfolio vector to satisfy the budget constraint (3.1) and the second-period endowment vector to satisfy the first part of the first-order conditions (3.4). Thus, we define

$$
\begin{align*}
\theta_{0i} &= (\bar{\theta}_{0i}, \hat{\theta}_{0i}), \quad \theta_{1i} = (\bar{\theta}_{1i}, \hat{\theta}_{1i}), \quad \theta_{ei} = \hat{\theta}_{ei} \\
c_i &= \hat{c}_i, \quad \lambda_{0i} = \bar{\lambda}_{0i}, \quad \omega_{2i} = \hat{\omega}_{2i},
\end{align*}
$$

(A.1)

where we introduced the following notation. The vector $\bar{\theta}_{0i} \in \mathbb{R}^{\Sigma - 1}$ consists of the first $\Sigma - 1$ components of $\theta_{0i}$ and $\hat{\theta}_{0i}$ is the last component of $\theta_{0i}$. The vector $\bar{\theta}_{1i} \in \mathbb{R}^{S - \Sigma}$ consists of all components, $\theta_{1is}(\sigma)$, of $\theta_{1i} \in \mathbb{R}^{SS}$ for which $s \in S(\sigma)$ and $s \neq \max S(\sigma)$. The vector $\hat{\theta}_{1i} \in \mathbb{R}^{S(S - 1) + \Sigma}$ consists of the remaining components of $\theta_{1i}$. Notice that $S(S - 1)$ components of $\hat{\theta}_{1i}$ are zero. The vector $\hat{\theta}_{ei} \in \mathbb{R}^{SJ}$ is simply $\hat{\theta}_{ei} = (1/I, \ldots, 1/I)$. All consumption and second-period endowment variables are hat-variables to indicate that they are all dependent variables. The Lagrange multiplier is a bar-variable to indicate that it is an independent variable. Overall, this introduces $(I - 1)((\Sigma - 1) + (S - \Sigma) + 1) = (I - 1)S$ independent variables.

For the last agent, $i = I$, we will use the consumption vector to satisfy the last part of the budget constraint (3.1), the second-period endowment vector to satisfy the first part of the first-order equations (3.4), and the security portfolio vector to satisfy the security market clearing condition.
Thus, we define

\[
\theta_{0i} = \hat{\theta}_{0i}, \quad \theta_{1i} = \hat{\theta}_{1i}, \quad \theta_{e1} = \hat{\theta}_{e1}
\]
\[
c_i = \hat{c}_i, \quad \lambda_{0i} = \tilde{\lambda}_{0i}, \quad \omega_{2i} = \tilde{\omega}_{2i},
\]

indicating that all portfolio choices are dependent variables. Thus, for agent \( i = 1 \) we have only 1 independent variable (the Lagrange multiplier). Notice that \( S(\Sigma - 1) \) components of \( \tilde{\theta}_{1i} \) are zero.

There are \( S + \Sigma \) non-zero security prices (non-zero components of \( (q_0,q_1) \)) and \( \Sigma J \) stock prices, \( q_e \). After taking into account the \( \Sigma + 1 \) normalization constraints for security prices, we are left with \( S - 1 \) security prices and \( \Sigma J \) stock prices, which will be used as independent variables. For these remaining prices, we assume strict positivity. The \( (S - 1 + \Sigma J) \) dimensional vector of independent asset prices will be denoted by \( \tilde{q} \in \mathbb{R}_{++}^{S-1+\Sigma J} \).

We will use the productivity parameters to satisfy the second part of the first-order conditions (3.4). More specifically, we write

\[
z_j = (\tilde{z}_j, \hat{z}_j),
\]

where the vector \( \tilde{z}_j \in \mathbb{R}_{++}^{S-\Sigma} \) consists of the components, \( z_j(s) \), of the vector \( z_j \in \mathbb{R}_+^S \) for which \( s \in S(\sigma) \) and \( s \neq \max S(\sigma) \). The vector vector \( \hat{z}_j \in \mathbb{R}^J_+ \) consists of the remaining components of \( z_j \in \mathbb{R}_+^S \). Overall, there are \( J(S - \Sigma) \) independent productivity variables. Adding up all the independent variables, we find that we have \( (I - 1)S + 1(S - 1 + \Sigma J) = IS + JS \) of them.

Let \( \tilde{e} = (\tilde{\theta}, \tilde{\lambda}_0, \tilde{q}, \tilde{z}) \) and \( \hat{e} = (\hat{c}, \hat{\theta}, \hat{\omega}_2, \hat{z}) \). Fix a point \( e^* = (\tilde{e}^*, \hat{e}^*) \in E_f \) and denote an open neighborhood of \( e^* \) by \( V_{e^*} \subset E_f \) and a relative open neighborhood of \( \tilde{e}^* \) by \( U_{\tilde{e}^*} \). The equalities (3.1), (3.2), and (3.4) implicitly define a function \( \tilde{\varphi} : U_{\tilde{e}^*} \rightarrow E_f \) in the following way.

The second part of the first-order conditions,

\[
\forall j, \sigma : \sum_s q_{1s}(\sigma)z_j(s) = 1,
\]

defines functions \( \hat{z}_j = \hat{z}_j(q_0, \hat{z}_j) \) since for each \( j \) and \( \sigma \) we can explicitly solve for the dependent variable \( \hat{z}_j(\sigma) \) (recall that for each \( \sigma \) there is at least one \( s \in S(\sigma) \)).

For the first \( i = 1, \ldots, I - 1 \) agents, the first part of the budget constraint,

\[
\forall i = 1, \ldots, I - 1 : \sum_{\sigma} q_{0\sigma} \theta_{0i\sigma} = 0
\]
\[
\forall i = 1, \ldots, I - 1 \forall \sigma : \sum_s q_{1s}(\sigma)\theta_{1is}(\sigma) + \sum_j q_{ej}(\sigma)\theta_{eij}(\sigma) = \theta_{0i\sigma} + \omega_{1i},
\]

defines functions \( \tilde{\theta}_i = \tilde{\theta}_i(\tilde{\theta}_i, \tilde{q}) \) since we can again explicitly solve for the dependent variables \( \tilde{\theta}_{0i\sigma} \) and \( \tilde{\theta}_{1is}(\sigma) \). Again for the first \( i = 1, \ldots, I - 1 \) agents, these functions in conjunction with the first part of the first-order conditions and the last part of the budget constraint,

\[
\forall i = 1, \ldots, I - 1 \forall \sigma, s : \frac{1}{\lambda_{0i}} \pi(\sigma, s)u'(c_i(\sigma, s), s) = q_{0\sigma}q_{1s}(\sigma)
\]
\[
c_i(\sigma, s) = \omega_{2s}(s) + \theta_{1is}(\sigma) + \sum_j z_j(s)q_{ej}(\sigma)\theta_{eij}(\sigma),
\]

The notation \( \tilde{z}_j = \tilde{z}_j(q_0, \hat{z}_j) \) is short-hand for \( \tilde{z}_j : \mathbb{R}^{S-\Sigma} \times \mathbb{R}^{S-\Sigma} \rightarrow \mathbb{R}^\Sigma, (q_0, \hat{z}_j) \rightarrow \tilde{z}_j(q_0, \hat{z}_j) \).
implicitly defines a function \( \hat{\omega}_{2i} = \omega_{2i}(\hat{\theta}_i, \lambda_{0i}, \bar{q}, \bar{z}, \hat{\theta}_i(\bar{q}, \bar{z}) \hat{z}(\bar{q}_1, \bar{z})) \). This follows from the assumption of strictly positive marginal utility and the fact that there are only \( S \) first-order conditions in (3.11) with \( r(\sigma, s) > 0 \), which completely separate. Of course, the second equation in (A.6) then defines a function \( \hat{\omega}_i = \hat{c}_i(\hat{\theta}_i, \bar{q}, \bar{z}, \hat{\theta}_i(\bar{q}, \bar{z}), \hat{z}(\bar{q}_1, \bar{z})) \) for every \( i = 1, \ldots, I - 1 \).

For agent \( I \), the market clearing conditions,

\[
\sum_i \theta_{0i} = 0 \\
\forall \sigma : \sum_i \theta_{1i}(\sigma) = 0, \sum_i \theta_{ei}(\sigma) = 1, \sum_j k_j(\sigma) = \sum_i \omega_{1i},
\]

(A.7)

define a function \( \hat{\theta}_I = \hat{\theta}_I(\bar{q}_I, \bar{z}_I, \hat{\theta}_I(\bar{q}_I, \bar{z}_I), \hat{z}(\bar{q}_1, \bar{z})) \) since we can explicitly solve for the portfolio holdings of the last agent. Using this function and the functions \( \hat{z}_j = \hat{z}_j(\bar{q}_1, \bar{z}) \) defined above, the first part of the first-order conditions in conjunction with the last part of the budget constraint,

\[
\forall \sigma, s : \frac{1}{\lambda_{0I}} \pi(\sigma, s)u'_I(c_I(\sigma, s), s) = q_{0e}q_{1s}(\sigma)
\]

\[
c_I(\sigma, s) = \omega_{2I}(s) + \theta_{1I}(\sigma) + \sum_j z_j(s)q_{ej}(\sigma)\theta_{ej}(\sigma).
\]

(A.8)

implicitly define a function \( \hat{\omega}_I = \hat{\omega}_I(\hat{\theta}_I, \lambda_{0I}, \bar{q}, \bar{z}, \hat{\theta}_I(\cdot), \hat{z}_I(\cdot)) \). This function in conjunction with the second part of (A.8) then defines a function \( \hat{c}_I = \hat{c}_I(\hat{\theta}_I, \bar{q}, \bar{z}, \hat{\theta}_I(\cdot), \hat{z}(\cdot)) \). The first part of the budget constraint (3.4) of agent \( I \) is automatically satisfied because of Walras’ law. Finally, goods market clearing follows from asset market clearing and the individual budget constraints.

So far, the argument neglected the non-negativity constraints on prices, multipliers, and endowments as well as the full-revelation condition (3.5). By assumption, these inequalities are satisfied for \( e^* \). By making the neighborhood \( U_{e^*} \) sufficiently small, we can ensure that these inequalities are also satisfied for all \( e \in V_{e^*} \). For \( \hat{\omega}_I > 0 \), this follows from the continuity of the functions \( \hat{\omega}(\cdot) \) defined above.

The function \( \hat{\phi} : U_{e^*} \to E_f \) is the composition of smooth functions and therefore smooth. By construction, it is also injective. The function \( \phi : U_{e^*} \to V_{e^*} \), \( \phi(e) = \hat{\phi}(e) \), \( V_{e^*} = \hat{\phi}(U_{e^*}) \) is therefore bijective. Smoothness of the inverse function, \( \phi^{-1} \), is obvious. Hence, \( \phi : U_{e^*} \to V_{e^*} \) is a local parameterization of \( E_f \). Since \( \dim U_{e^*} = SI + SJ \), the set \( E_f \) is a smooth manifold of dimension \( SI + SJ \).

Consider now the set \( E_p \). Notice first that the set \( E_p \) is the union of \( \Sigma(\Sigma - 1)/2 \) sets, namely the sets \( E_p(\sigma, \sigma') \) defined for \( \sigma \neq \sigma' \) as

\[
E_p(\sigma, \sigma') = \{(c, \theta, \lambda, q, \omega_2, z) \in E_p | q_{ej}(\sigma) = q_{ej}(\sigma') \forall j \}.
\]

For each \( E_p(\sigma, \sigma') \) we can use a parameterization similar to the one for \( E_f \), with the exception that we add the restrictions

\[
\forall j : \quad q_{ej}(\sigma) = q_{ej}(\sigma)
\]

(A.9)

Since in our parameterization of \( E_f \) equity prices are free variables, adding the restriction (A.9) is straightforward. The equalities (3.12), however, reduce the number of free parameters by \( J \). Thus, each \( E_p(\sigma, \sigma') \) is a smooth manifold of dimension \( SI + SJ - J \), from which the claim follows. \(^{12}\)
Proof of lemma 2.

(i) Emptiness of $\text{FCE}_p(\omega_2, z)$ and non-emptiness of $\text{FCE}_f(\omega_2, z)$.

For any signal pair $\sigma \neq \sigma'$ consider the projection $\text{proj}_{p\sigma'}: \text{proj}_{p\sigma'}(c, \theta, \lambda_0, q, \omega_2, z) = (\omega_2, z)$. From the proof of lemma 1 we know that $\text{proj}_{p\sigma'}(\sigma, \sigma')$ is a smooth manifold. Clearly, the projection mapping is smooth. Hence, by Sard’s theorem, the set of regular values of the projection mapping is of full Lebesgue measure. Since $\text{proj}_{p\sigma'}(\sigma, \sigma')$ has dimension $SI + SJ - J$ and $\Omega \times Z$ has dimension $SI + SJ$, the regular value theorem implies that the preimage set, $\text{FCE}_{p\sigma'}(\omega_2, z) \equiv \text{proj}_{p\sigma'}^{-1}(\omega_2, z)$, is empty for generic $(\omega_2, z) \in \Omega^* \times Z^*$. Thus, for generic $(\omega_2, z) \in \Omega^* \times Z^*$ the set $\text{FCE}^*(\omega_2, z)$ is the union of a finite number of empty sets, and therefore empty. Since for any $(\omega_2, z) \in \Omega \times Z$ the set $\text{FCE}(\omega_2, z)$ is non-empty, we conclude that for generic set $(\omega_2, z) \in \Omega^* \times Z^*$ the set $\text{FCE}_f(\omega_2, z)$ is non-empty (recall that $\text{FCE} \equiv \text{FCE}_p \cup \text{FCE}_f$).

(ii) Finiteness

Consider the projection mapping $\text{proj}_f: E_f \rightarrow \Omega \times Z$, $\text{proj}_f(c, \theta, \lambda_0, q, \omega_2, z) = (\omega_2, z)$. From lemma 1 we know that $E_f$ is a smooth manifold of dimension $SI + SJ$. Clearly, the projection mapping is smooth. Hence, by Sard’s theorem the set of regular values of the projection mapping has full Lebesgue measure, that is, the set of regular values is generic set $\Omega^* \times Z^*$. Since both the domain and the image set of the projection map are smooth manifolds of the same dimension, $SI + SJ$, the regular value theorem implies that the set $\text{FCE}_f(\omega_2, z) \equiv \text{proj}_f^{-1}(\omega_2, z)$ is finite for all $(\omega_2, z) \in \Omega^* \times Z^*$. □

Proof of lemma 3. Clearly, the set of fully-revealing equilibria for which $c_i(\sigma, s) = c_i(\sigma', s)$ \forall i, $\sigma, \sigma'$, $s$ is a subset of the set of fully-revealing equilibria with $\sum_i c_i(\sigma, s) = \sum_i c_i(\sigma', s)$ for some $\sigma, \sigma'$ $s$. We show that the latter set is empty for a generic set of economies. To do this, note that the goods market condition together with $\sum_i c_i(\sigma, s) = \sum_i c_i(\sigma', s)$ \forall $\sigma, \sigma', s$ imply $\sum_j z_j(s)q_{ej}(\sigma) = \sum_j z_j(s)q_{ej}(\sigma') \forall \sigma, \sigma', s$ (the endowments are signal-independent). In our parameterization of the equilibrium manifold used to prove lemma 1, the equity prices are free parameters. Thus, we can use one of those prices to ensure that the equality constraint with $\sum_i c_i(\sigma, s) = \sum_i c_i(\sigma', s)$ holds for some $\sigma, \sigma', s$, and otherwise repeat the argument used in lemma 2. This shows that the resulting equilibrium manifold has a dimension strictly smaller than the dimension of $E_f$. Thus, applying an argument similar to the one made in the proof of lemma 2, we conclude that for a generic set of economies the equality constraint $\sum_i c_i(\sigma, s) = \sum_i c_i(\sigma', s)$ is violated. □

References


